MECHANICAL DRAWING.

PROGRESSIVE EXERCISES

AND

PRACTICAL HINTS.

FOR THE USE OF ALL WHO WISH TO ACQUIRE THE ART, WITH OR WITHOUT THE AID OF AN INSTRUCTOR.

BY

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PREFACE TO PART I.

It is proper to state that the second part of this book was separately published some time before the first part was written; which accounts for the fact that the chapter on Drawing Instruments is found at the end, and not at the beginning, where it might naturally be looked for.

The exercises contained in Part I embrace the essential features of a course of instruction which has been successfully pursued for twenty years. Those given in the first two chapters are intended not only to train the eye and the hand in the use of instruments, but also to form the habit of exercising forethought, judgment, and taste in relation to the important matter of arrangement.

Reasonable skill in execution having been acquired, the next step is to the delineation of solid objects. And in the treatment of projections the aim is to lead the student to draw these in a matter-of-fact way, as they would appear from different points of view; the express design being to keep out of sight as far as possible the artificial and often useless stage machinery of Descriptive Geometry.

In this manner the beginner is enabled almost at once to construct intelligently simple detail drawings that have a practical meaning and a use, which of itself is a great incentive to further efforts. But this is not the only nor the strongest reason for adopting this course. There are many to whom the representations of tangible and actual things are perfectly clear, while those of abstractions, such as points, lines, or planes, in space, are obscure if not unintelligible; nevertheless they may and often do become expert and valuable practical draughtsmen.

But there are none who will not find a thorough familiarity with the former to be of the greatest advantage in the study of the latter,—one does not study the structure of language before he begins to use it,—and in this subject, as in many others, some knowledge of the concrete is essential to a ready apprehension of the abstract.

C. W. MacCord.

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MECHANICAL DRAWING.

CHAPTER I.

ELEMENTARY EXERCISES: STRAIGHT LINES AND CIRCLES.

1. **Mechanical** drawing and **geometrical** drawing are by some writers used as synonymous terms; in many treatises the subject is opened up by the explanation of various geometrical problems, and the first exercises consist in drawing, according to the explanation, diagrams representing their solution—thus combining elementary instruction in the science of geometry with that in the art of drawing.

No doubt a knowledge of that science is essential to proficiency in the art; but it is not clear that to study both at once is the best way to acquire either. Those to whom the following pages are addressed, then, are presumed to have gained from other sources an acquaintance with such geometrical principles as may be made use of in the course of the work.

2. A single illustration will serve to show that a distinction may justly be made between the terms above specified.

Let it be required in Fig. 1 to draw through P a perpendicular to AB.

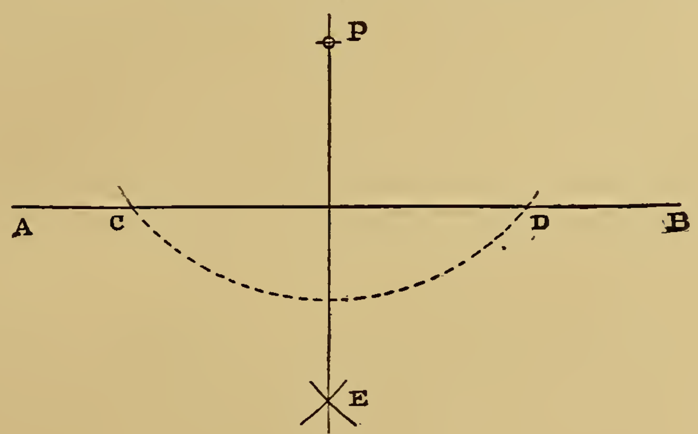

FIG. 1.

The geometrical process is as follows:

1. Describe about P with any assumed radius a circular arc cutting AB in the points C and D.

2. Describe about C and D, with equal radii assumed at pleasure, two other arcs intersecting in E.

3. Draw the straight line PE; it is the perpendicular required.

Now if in the above or any other case, the required result is attained by actually executing the geometrical processes, the operation may properly be called **geometrical** drawing. And it is to be noted that the only instruments admissible are the dividers or compasses, and the ruler or straight-edge. The methods are precise, but the manipulations are often tedious.

3. But by adding a few other simple implements, many operations of constant occurrence in making industrial drawings which involve geometrical principles can be performed with far greater facility, and yet with all the accuracy needed for practical purposes. In illustration may be mentioned the drawing of horizontal and vertical lines by means of the T-square; the use of the scale for bisecting and subdividing lines; and the use of the triangles for drawing lines parallel or perpendicular to others in any position, for inscribing or circumscribing certain polygons, as well as for drawing tangents to circles and finding the points of contact. And that may appropriately be called **mechanical** drawing, in which the use of these and any other labor-saving appliances or devices is permitted. As a matter of fact, they are used in the execution of by far the greater part of the drawings made for constructive purposes.

4. Accepting the latter definition, the following exercises are selected and arranged with a view to imparting facility and accuracy in the use of the instruments. Before proceeding to consider them, it may be well to give a few hints as to the use of the pencil; it is important to begin right, as habits once formed are not easily changed.

1. **Pencil lightly.** Outlines to be inked in should, however, be distinct and firm; centre lines and the like should be finer than these, while "construction lines," etc., which are not to be inked in, should be the finest of all.

2. **Pencil clearly.** Keep the ruling pencils and the instrument leads sharp. Leave in no superfluous or "false" lines; if a line is by any oversight drawn in erroneously, do not leave it to be rubbed out after inking in the rest, but *erase it at once.*

3. **Pencil in only what is to be inked in,** whenever practicable; it is easier to leave out lines, or parts of lines, than to rub them out: and the neatest draughtsman is the one who does the work with the least use of either pencil or rubber. Also, lines which are to be dotted in ink should be dotted in pencilling.

4. Whenever it is possible, mark the extremities of a line by means of the scale, *before drawing the line.* This is often much facilitated by placing the scale against the edge of the T-square; and it enables us to draw the line at once of its exact length, in compliance with the preceding suggestion.

5. In setting off measurements, and in subdividing lines, make use of the scale whenever it can be done, and use the dividers or compasses only when it is unavoidable.

6. Do as much as possible with one setting or adjustment of any instrument, particularly with the dividers or compasses. Much time may be wasted by neglect of this simple precept.

5. In short, the drawing should be made in pencil as nearly as possible as it is to be made in ink. It may take more time to execute the pencilling, but when the habit has been once formed, the difference is less than might be imagined, and is often nearly balanced by the saving of time in the inking in. A chief draughtsman may wish personally to lay out work, which he can, if thus executed, safely leave to be inked in by his assistants; and again, a drawing thus made can upon occasion be traced at once, without being put in ink at all, by which a material saving of time may be effected.

Having mentioned the element of time, it may be well here to add a caution against undue haste, a frequent cause of oversights and errors, which is apt to lead to careless or slovenly work. "*Make haste slowly*" is the golden rule of the draughtsman, whose most valuable qualification is **accuracy**; the next is **neatness**; and the third in importance is **speed**, which is valuable only in so far as it is the result of skill and confidence gained by careful practice, —and worse than worthless if acquired at the expense of reliability.

6. It is recommended for the sake of neatness and uniformity that the beginner should make a series of sheets measuring 19 inches by 12, a size requiring a quarter sheet of Double Elephant paper. A margin of one inch all around leaves a rectangle of 17 inches by 10, to be enclosed by a plain or ornamental border. Within this the drawings forming the exercises are to be arranged, with such symmetry as the subjects may admit; and made of such dimensions that they shall neither be so small as to seem lost in the surrounding space, nor so large as to appear crowded.

The effort to accomplish this gives from the outset a training in judgment and forethought which could not be hoped for were exercises of the same nature gone through with regardless of size or of external limitations.

7. The sheet should be placed as near as may be to the lower left-hand corner of the drawing-board, and may be secured by means of four drawing-pins. But, *two* pins will always suffice to keep the paper in place, and one or more must be temporarily removed if they interfere with the T-square or triangles, which should always lie *flat on the paper*. This being arranged, the outer lines may be drawn from edge to edge of the paper, since the sheet when finished is to be cut to the size thus indicated. This final trimming should always be done with *shears;* and never with a knife, the use of which would injure the drawing-board, and sooner or later ruin the T-square, if, as is too likely, it were used as a ruler for this purpose.

8. Fig. 2 represents a completed sheet before trimming; and the exercises which it contains are such that by due attention to the preceding suggestions it is easy to finish up the whole without using the rubber at all. Thus, having set off a point one inch below the upper line, set the T-square against the left side of the drawing-board, and place the scale along its upper edge. Slide the T-square up until the upper edge of the scale passes through the point; then sliding the scale along the square, set off a point one inch inside of each of the outer vertical lines, thus locating the upper corners of the border. Repeating this at the lower side of the sheet, the border can be drawn of its exact size. Again, mark a point one inch below the upper line of the border, and using the T-square and the scale together as before, slide the latter along until its zero point is on the left-hand line of the border, and with the pencil mark the distances 1, 5, $6\frac{1}{2}$, $10\frac{1}{2}$, 12, and 16, *without moving the scale*. These points locate the upper corners of the three rectangular figures drawn in the upper part of the sheet; and it need not be described in detail how in a similar manner all the corners of all the rectangles, as well as the centre lines of the circles drawn in two of them, may be located, so that all the pencil-lines may be drawn without "overrunning" or intersecting.

9. Particular attention is called to the manner of using the scale above mentioned, viz., the setting off *of the sums of successive distances* from a fixed zero. A very common error is to set off the first, then to move the scale, and placing the zero at the point thus marked, to set off the second, and so on. Since there is always a possibility of making an error in

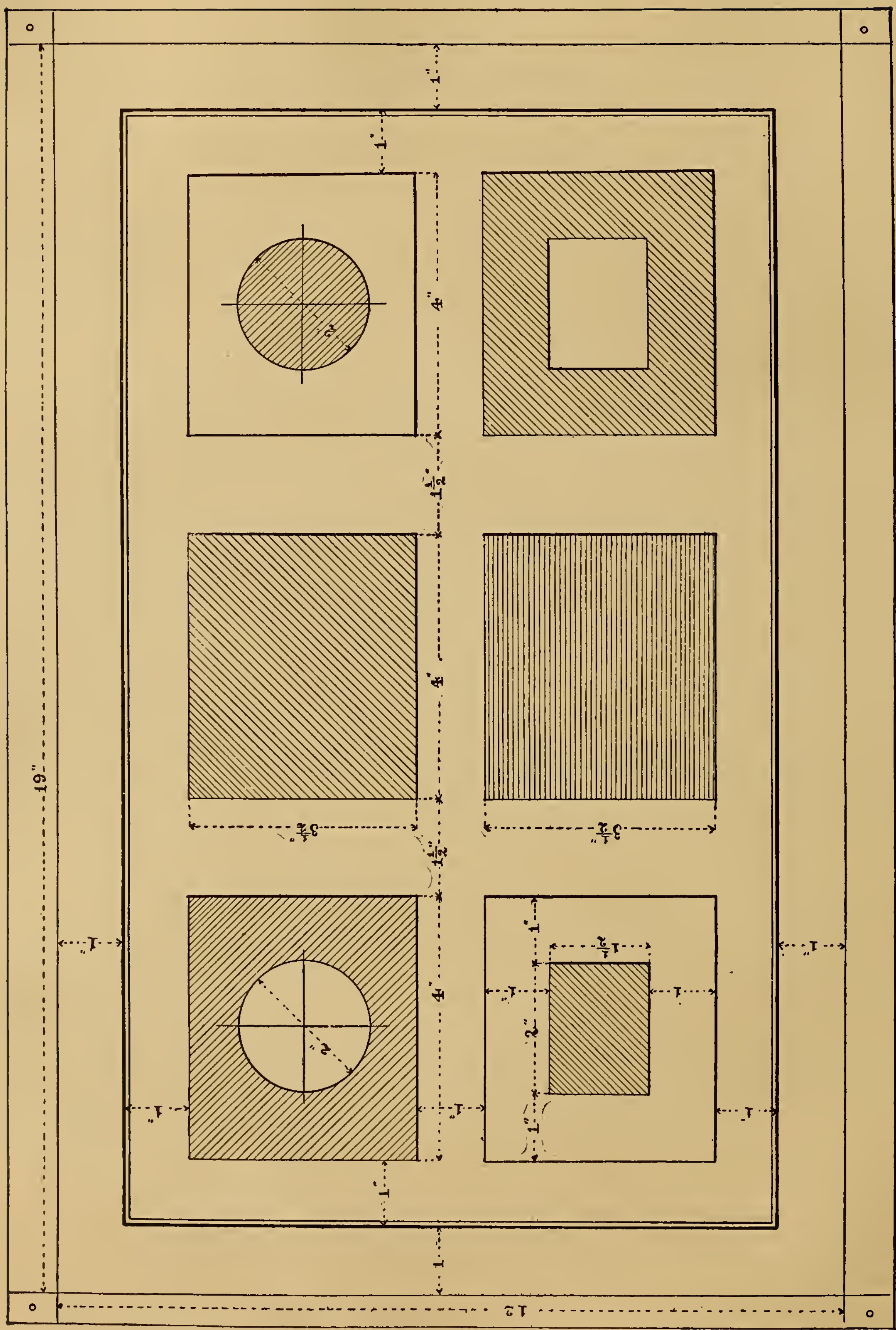

Fig. 2.

setting off any distance whatever, the result of this method is likely to be that the total distance is not exactly equal to the sum of its parts, because the errors may accumulate. In adopting the method recommended, there is at least the security that the total error in laying off any distance within the limit of the scale will not exceed that which may exist in the measurement of one of the parts. This is of greater importance in making working drawings, but a correct method should be used from the very beginning.

10. In regard to "inking in," the following general instructions apply in all cases when the best results are desired:

1. Do not take the pen in hand until the pencil is done with; then ink in all the outlines, of uniform thickness, drawing the circles and circular arcs, if there are any, first, and leaving the straight lines till the last.

2. The centre-lines and construction-lines, if there are any, are to be put in next; and these should be "hair-lines," i.e., the finest of all upon the sheet.

3. When this is done the sheet should be cleaned, and all traces of pencilling removed.

4. The dotted "dimension-lines," if any, should now be put in, and the arrow-heads and figures next.

5. If any parts are shown in section, the "hatching" or "sectioning," by which this is indicated, should be done next.

6. If there be any "line-shading," by which the forms of uncut surfaces are indicated, this should be now executed.

7. The "shadow-lines," which indicate the edges that intercept the light and cast shadows, are put in last of all.

Some terms not yet explained have necessarily been introduced in the above: these will be considered in due course.

11. Thickness of Lines.—Lines are distinguished as fine or coarse, thin or thick, light or heavy, indifferently. The absolute thickness of line which is suitable in any given case cannot be specified by a positive rule, but must be learned by observation and experience. In a general way, however, it may be stated that the smaller the objects represented, and the smaller the scale upon which they are drawn, the finer the lines should be. But it is quite clear that the thickness cannot vary precisely in proportion to the scale, because drawings made on even the smallest scales must be defined by lines which are distinctly visible. Whether thick or thin, however, the lines should be smooth, even, and continuous, and if in ink, perfectly black; as the finish of a mechanical drawing depends very largely upon the quality of the lines.

12. Junction of Lines.—When two lines meet and terminate at the same point, as for example in the corners of the rectangles in Fig. 2, especial care must be taken to make the junction sharp and clean, as shown in the illustrations on the left, in Fig. 3. If through haste or carelessness one or both the lines be made too long or too short, as illustrated on the right, the effect is exceedingly bad, no matter how smooth the lines. Minute defects like this are the first to catch the eye of the expert, while more important errors might escape notice; and they hold the eye, too, like little flaws in a mirror or little cracks in a vase. Indeed, the finish of a drawing depends much upon such small and apparently insignificant things; and besides, it is a fair inference that one who is careless about these points of detail

may be so in regard to greater matters. These remarks apply with equal stringency to the plain "border-line" around Fig. 2, which of itself is, no less than the figures contained

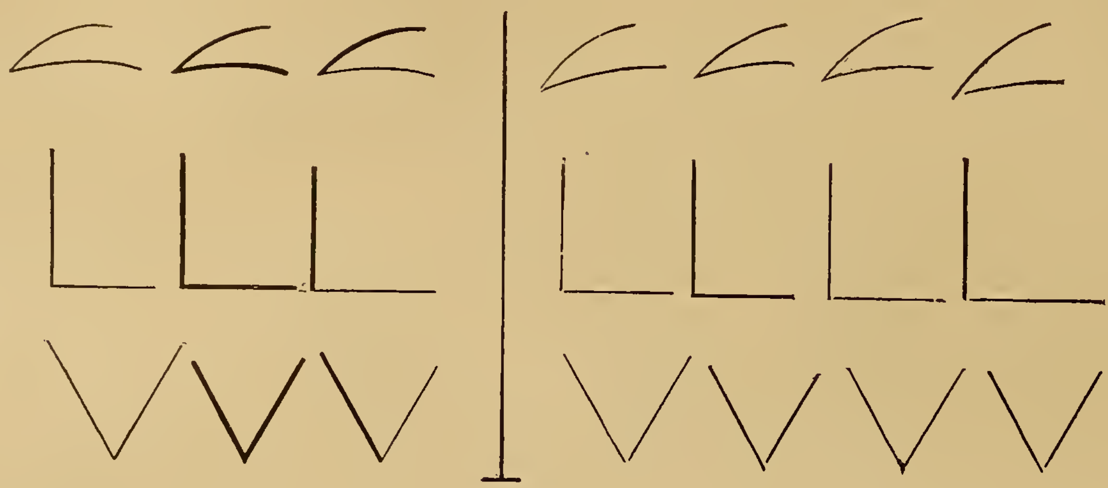

FIG. 3.

within it, an exercise in mechanical drawing; it should be treated with equal deference, and drawn with the same care.

13. Sectioning.—When an object or any portion of it is supposed to be cut by a plane, as in mechanical structures is often necessary in order to show the interior arrangement, the surface of the cutting plane is conventionally indicated by "hatching," to which reference was made in (**10**). This consists merely in covering the surface with fine parallel lines, making ordinarily an angle of 45° with the bounding-lines of the surface when these are straight, and an angle of 45° with the horizontal line when the outlines are curved; which inclinations, however, are subject to modifications in cases to be subsequently discussed. Various parts of the exercise in Fig. 2 are shown as thus "hatched," or, as it is more commonly called, "sectioned," which will serve to give practice in the use of the pen and also of the triangles, meantime training the eye also in judging of equal spaces; for it is to be understood that the distances between the lines are not to be spaced off by any instrument, but determined by the eye alone. The operation consists simply in ruling one line after another, sliding the triangle along the edge of the T-square for the same distance after drawing each one.

14. When a few lines have thus been drawn in succession, an unconscious rhythmic action, as it were, is established, just as one beats time to fast or slow music without thinking, and the manipulation becomes to a great extent mechanical. For this reason a drawing should never be sectioned in pencil first, but this should be done in ink at once: the pencilled sectioning may be excellent, but if inked in afterward, the result is likely to be as bad as if one were to write his name with a pencil and then try to go over the lines with a pen. And for the same reason the sectioning of a drawing should, whenever it is possible, be done at one continuous operation; for if a part be done one day, and another the next, there is likely to be a perceptible difference in the *tone* of the two parts. And again, in making a tracing from a completed drawing no attempt should be made to trace the section-lining: the outlines having been copied, a sheet of white paper is slipped under the tracing-cloth, and the sectioning is done as upon a new drawing.

15. No absolute rule can be laid down as to the width of the spaces between the lines. The general effect to be aimed at is that of a light tint covering the surface; hence it is clear that **the lines of the sectioning should be finer than the outlines.** If a narrow surface be sectioned with wide spaces, the effect will be coarse and unpleasing; *per contra,* the larger the surface, the wider the spaces may be made. If the spaces be too narrow, the effect will be that of a dark tint, tending to crush out the outlines: and besides, it not only requires more time to do the work, but any error in the spacing will be much more conspicuous; and the evenness of the tone evidently depends upon the uniformity of the spaces. It is to be noted also that the section-lines must begin and end at just the right points, neither overrunning nor falling short of the outlines; otherwise the effect is rough and ragged.

16. But the evenness of the tone also depends upon the uniformity of the lines in both tint and thickness. If the ink thickens in the pen, the result is a line either too dark or too thick, or both. The so-called "prepared ink" is peculiarly liable to become viscid in the pen, thus producing this exasperating result; and there is not a bottle of it, of any make, with which it would be safe to undertake the sectioning, much less the line-shading, of any elaborate plan. **Throw the bottle out of the window**; mix up a liberal quantity of good China ink with clear water in a clean saucer, a little thinner than for outlining; test the pen; practise a minute on some scrap of paper until hand and eye act in unison and the proper space is determined; then proceed with confidence and hope of success. In Fig. 2 one of the rectangles is ruled with equidistant horizontal lines, in the manner of line-shading a plane surface. These, too, are to be spaced by the eye, and the above remarks apply to them as well.

17. Shadow-lines.—In Fig. 2 it will be observed that some of the outlines are much heavier than others; and no one can fail to be struck by the effect of *relief* thus produced, nor to appreciate that the figures are not mere diagrams, but represent objects of sensible thickness, which if resting upon a plane surface would cast shadows upon it. The lines which are thus made heavy are technically called **shadow-lines**; and in all cases they represent the sharp **edges** of surfaces, which intercept the light. Thus they not only give the power of producing relief in outline drawings, but afford at once the means of distinguishing an object which has sharp edges, as for instance a prism, from one which has not, such as a sphere or a cylinder. Of course it would be out of place here to attempt the full explanation of how to determine in all cases what lines do cast the shadows. But this must depend to a great extent upon the direction from which the light comes; and for the sake of uniformity it has been agreed that on the drawing this direction shall in all cases appear to make an angle of 45° with the horizontal line, as indicated by the arrow in Fig. 4, it being understood that the rays of light are not *parallel* to the paper, but come from a source above the object as well as to the left of it.

18. Suppose then that in this figure *ABCD* represents a square block lying on the paper, and perforated by a square opening *EFGH*, all the edges being sharp. It will be seen at once that *AB*, *BC*, *EH*, and *HG* are the lines which cast shadows, and they are made heavy accordingly. Let *X* represent a torus, or ring, such as can be made by bending a cylindrical rod around a mandrel: this body would certainly cast a shadow, but it has no edges, and therefore no shadow-lines Let *Y* represent an annulus, or ring formed by bending a square bar around a mandrel: then the outer and the inner circles represent edges, and portions of

both will cast shadows, falling away from the light. Draw rays of light tangent to the outer circle at *m*, *n*: then, as any one can readily satisfy himself by laying a disk, say a coin, upon the table in the sunlight, the semicircumference *man* will not cast a shadow, while *nbm* will do so; moreover, the shadow will be broadest opposite *b*, the point midway between *m* and

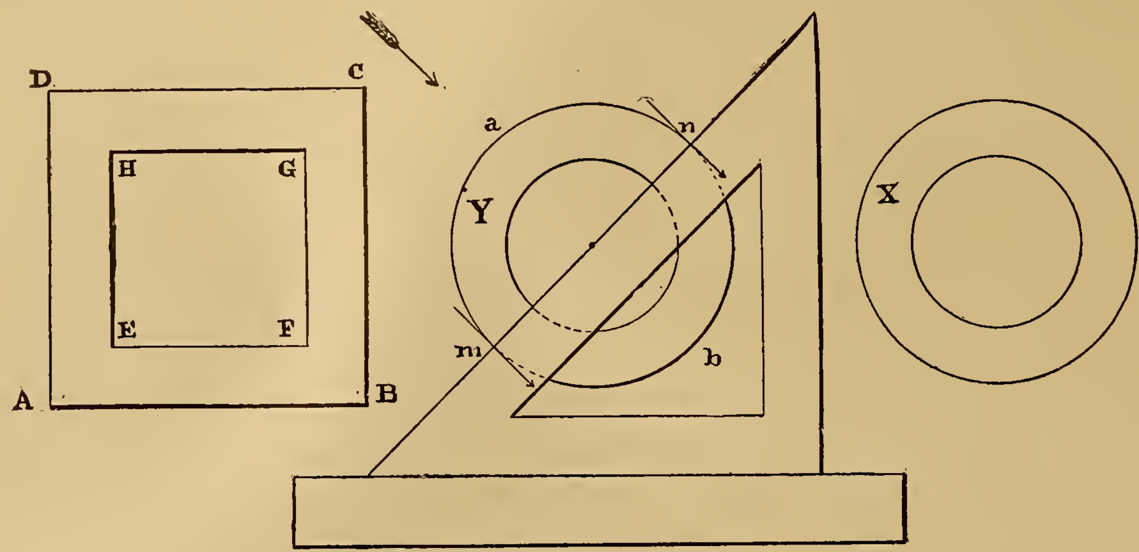

FIG. 4.

n, gradually diminishing in breadth as it approaches these points of tangency, where it will disappear. And, similarly, the opposite half of the inner circumference will cast a shadow, likewise tapering out to nothing at the points of tangency to parallel rays.

19. Of course it is not necessary actually to draw these rays, since, as indicated in the figure, the points of tangency can be at once marked by means of the 45° triangle.

In order to draw this tapering shadow-line the compasses must be put in motion *before the pen touches the paper* at *n*; the pressure is gradually increased as the pen approaches *b*, gradually diminished as it approaches *m*, and the pen is removed from the paper *while still in motion*. The whole action is very similar to that of free-hand flourishing with a writing-pen, and is easily acquired with a little practice.

If the line is to be quite heavy, it may be necessary to go twice or even three times over the broader part. Beginners are very apt to think that the object can be accomplished by slightly shifting the centre—a very dangerous expedient, unless the line is to be made excessively thick, as it might be for a lecture-room diagram; for it is almost impossible to move the needle-point through a distance sufficiently minute without having it slip back again into the hole made by it in the original position. For all ordinary purposes the method above advised is the safest and the best.

20. Border-lines.—The preceding instructions embrace all that is necessary for the completion of the *exercises* given in Fig. 2, and apply equally to all those following which come within their scope. A word may now be said as to the border-line by which they are surrounded. In general such an ornamental adjunct is not appropriate in connection with separate detail drawings; but when a series of sheets of uniform size is to be made, it may be introduced with good effect. Around topographical drawings, however simple and crude, a border of some kind is regarded as almost indispensable. But wherever used, it is in a

sense ornamental and superfluous, it challenges criticism, and should be executed with scrupulous care. Subsequently some examples will be given of more ornate borders with correspondingly elaborate "corner-pieces," suitable for drawings of a pictorial nature; for the present, the less pretentious "plain border," consisting, as shown in Fig. 2, of a fine interior line and a thicker one without, will suffice to exercise the skill of the tyro.

The absolute thickness of the outer line is to a great extent dependent upon individual taste; the standard adopted in topographical work is $\frac{1}{100}$ of the breadth of the map—which would give, for the borders here proposed, enclosing a space 10 inches wide, an outside line $\frac{1}{10}$ of an inch thick: and this should also be the breadth of the white space between the outer and inner lines.

No attempt should be made to make even a line so thick as this at one stroke of the pen, nor yet to cover the whole with ink *at once*, as is sometimes done by means of a brush, or worse yet, by "filling in" the space ruled by two finer lines by the use of either the drawing-pen or a writing-pen. Such a river of ink not only requires a long time to dry, but is apt to cause the paper to "buckle," after which it is very unlikely to become smooth upon drying.

The proper course is this: Rule parallel lines of moderate thickness *a short distance apart;* when these are thoroughly dry, and not before, rule others between them, until the whole breadth is covered. Thus a smooth, black, and even line of any width may be obtained.

21. Converging Lines.—When, as in Fig. 5, a number of lines are to be drawn radiating from a single point, it is clear that one will meet and merge into another at some little distance from the actual point of convergence. In such cases wait until each line is dry before drawing the next one, in order to avoid the accumulation of a mass of ink at the junction. And this will be avoided with the greater certainty by drawing the lines *away from* and not *toward* the point from which they radiate.

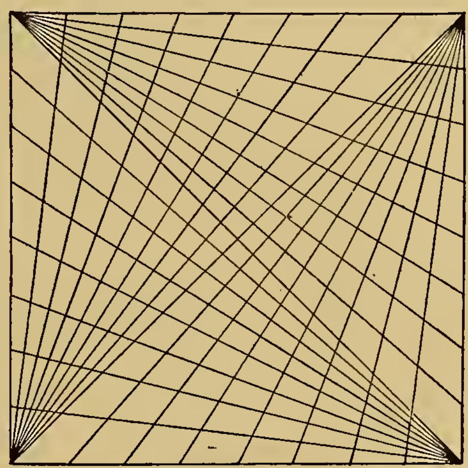

FIG. 5.

22. Fig. 6 is a sheet of exercises similar to those of Fig. 2, and no additional instructions are required, except in relation to the inner corners of the hollow rectangle, which are rounded, or, as it is technically called, "filleted." In pencilling, the sides of the inner rectangle are drawn first, and the circular arcs next drawn tangent to them with the bow-pencil. The centre c of such an arc, Fig. 7, A, can after a little practice be readily and accurately located by trial, after which, if it be deemed necessary, the points of contact, m, n, may be marked by aid of the T-square and triangle. If the radius of the circle is large, it will be advantageous to draw a short line at an angle of 45° through the corner x of the pencilled rectangle, upon which the centre must lie, as in Fig. 7, B; or to set off from x, as shown at E, the points m, n, with the given radius, and around these points to describe short arcs with the same radius, intersecting in the required centre.

But in drawings of machinery so many of these small fillets are met with, that such geometrical operations consume too much time.

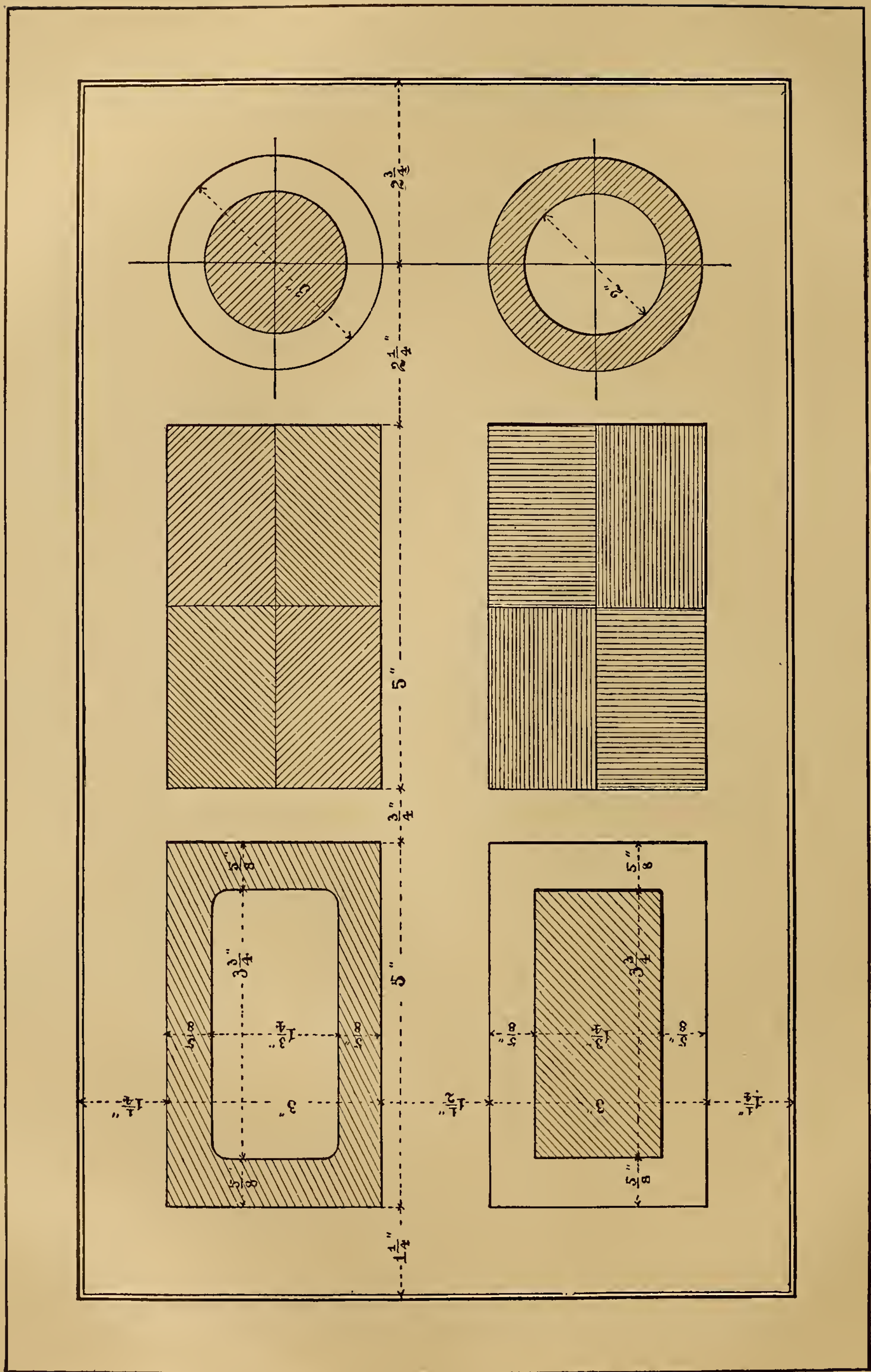

FIG. 6.

23. Ink in the circular arcs first and the right lines last, because it is much easier to join the right line smoothly to an arc already drawn, than to join an arc smoothly to even one right line, let alone two—as may be readily proved by trial. In the case of a circular arc the reason is clear, for the least error in either centre or radius will produce a sensible effect at one or both the junctions. It may be said that such an error would impair the accuracy of the work even if the arc were inked in first; this is true, but if the error be, as it often is, practically unimportant, there is no need to make it conspicuous, as it will be if the junction, as at Fig. 7, *D*, is careless or imperfect.

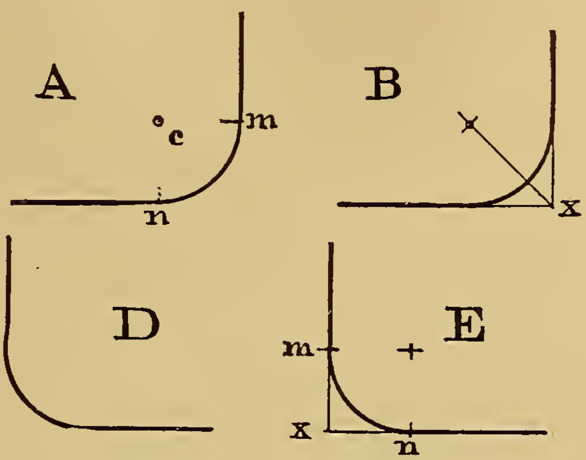

FIG. 7.

The difference in the difficulty of making a continuous line is not so great if the curve be one for which a "sweep," or curved ruler, can be used; but a systematic procedure is likely to save time, and straight-line work is preferably left to the last.

24. Sectioning of Hollow Figures.—In Fig. 8 are shown transverse sections of a hollow rectangular beam and a hollow cylinder. In sectioning these at the angle shown, it is natural and right to begin at *C*. After passing the point *E* many think it necessary to section the

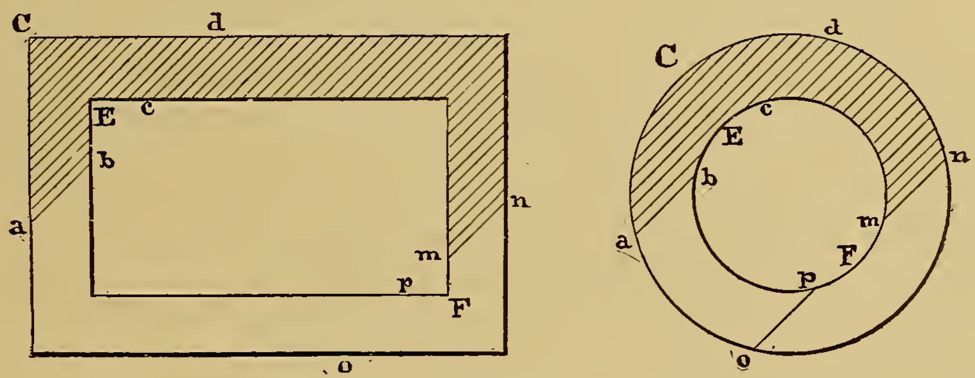

FIG. 8.

opposite sides *at the same time*, drawing, for instance, the line *ab*, then the line *cd*, without moving the triangle. This involves a serious loss of time, if the surfaces are of any considerable magnitude.

A better method is to section the upper side first, going on from *cd* to the right, stopping when a line *mn* is drawn, such that a half-dozen or so more would carry us to the point *F*. Then go back to *ab*, and section the other side—it will then be seen that when the edge of the triangle again coincides with *mn*, the error in the position of *op* cannot exceed half a space; and by dividing this error up among the spaces yet remaining before reaching *F*, the individual variations will be imperceptible. The saving of time in the sectioning of a large cylinder by adopting this method instead of the other amounts to fully twenty-five per cent, after a moderate degree of facility in the manipulation has been attained.

TANGENCY OF CIRCLES AND LINES.

25. For the purpose of acquiring skill in the handling of the compasses, and habits of care in the execution of work at every step as well as in the final processes, no practice is better than that afforded by exercises in which it is required to draw a number of circles within a given figure, tangent to it and to each other; some of which may now be appropriately taken up, in order to relieve the monotony of continuous straight-line work.

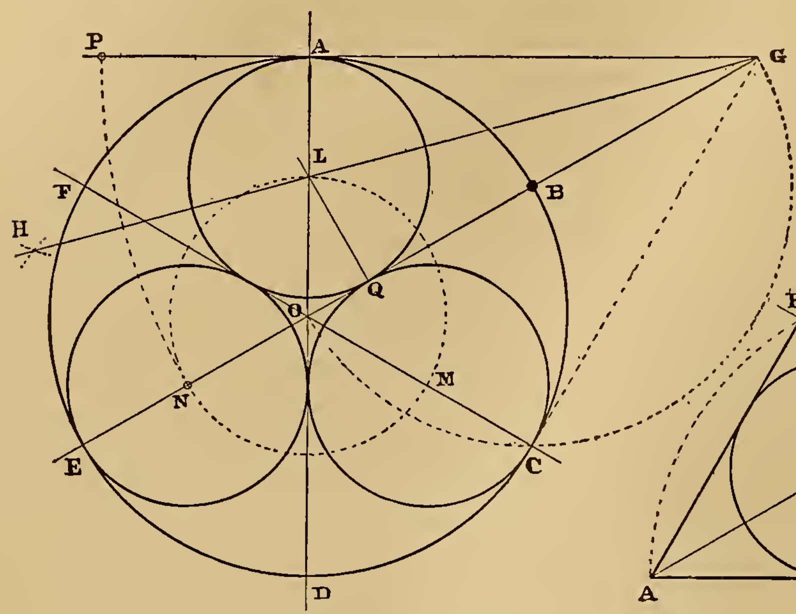

FIG. 9.

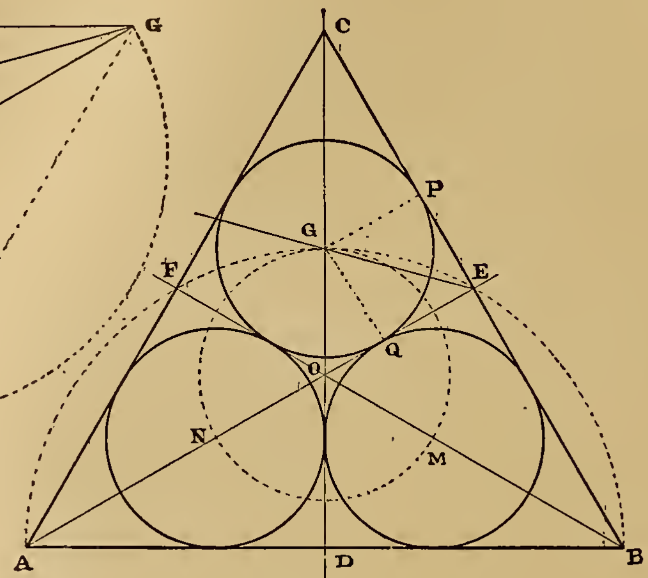

FIG. 10.

Two such exercises are given in Figs. 9 and 10, the external figures being a circle and an equilateral triangle, within each of which three tangent circles are to be inscribed.

26. To construct Fig. 9.

1. Draw the vertical centre line and describe the larger circle about *O*.

2. *Without laying down the compasses*, set the needle-point at *D*, and mark the points *C*, *E*, the radius being unchanged.

3. Similarly, set the needle-point at *A*, and mark the points *B*, *F*.

4. Still with the same radius, describe about *B* the semicircle *OCG*.

5. Draw the diameters *FC*, *EB*, and produce the latter to *G*.

6. Draw the tangent at *A*, and produce it also to *G*.

7. Bisect the angle *AGO* by the right line *GH*, cutting *AD* at *L*.

8. Describe about *O* a circle through *L*, cutting *CF* and *EB* at *M*, *N*.

9. About *L*, *M*, *N*, with radius *LA*, describe the three required circles.

27. To construct Fig. 10.

1. Draw the vertical centre-line of indefinite length.

2. Lay the scale against the T-square, set off $DB = DA$, and draw *AB*.

3. Set the zero of the scale at *A*, and mark the point *C* on the vertical centre-line, making $AC = AB$.

4. See if *CB* is equal to *AB*; and if correct, mark its middle point *E*.

5. Mark with the scale the middle point *F* of *AC*.

6. Draw AE, BF, cutting CD in O, the centre of the triangle.
7. Bisect the angle AEC by a line cutting CD in G.
8. Describe a semicircle on diameter AB: it should pass through G.
9. Describe about O a circle through G, cutting AE, BF, in N and M.
10. Draw GP perpendicular to BC; it is the radius of the required circles.

28. The above instructions have been made thus explicit in order to emphasize one or two points already mentioned; more particularly, the doing as much as possible at once with one adjustment of the compasses, as in Fig. 9, and the use of the scale when practicable instead of the compasses, as in Fig. 10. It is very common to see the triangle laid out by striking arcs about two of its angles as centres: in doing this, holes are pricked in the paper, into which the ink will run, so that the sharp finish desirable is dstroyed. **Use the scale.**

Another object is, to call attention to the advisability of *checking* the accuracy of any step, when the greatest precision is required. Thus in Fig. 9 the point G is the intersection of the tangent at A with the prolongation of the diameter EB; and the precision with which it is located is confirmed if it falls, as it should, upon the circumference OCG: evidently, an error here would be vital, and affect injuriously the final result. Another instance is found in Fig. 10, where the intersection of the two straight lines EG, CD should fall on the semi-circumference AGB. The application of such checks is particularly necessary in laying out diagrams of this description, for though geometrically simple, they are practically very difficult to draw with perfect precision.

29. Bisection of an Angle.—In Fig. 9 the arc NP is drawn, merely to indicate that the arcs intersecting at H, in bisecting the angle AGO, are described about centres P and N, equidistant from the vertex G; and thus in this illustrative diagram to make a record of the processes. Practically, the arc need not be drawn at all, it being sufficient to set off by the scale the two points P and N, and the farther from G the better; for the distance between H and G being thus increased, any error in the direction of the line determined by those points will diminish as the line approaches the vertex. Theoretically, the radius of the arcs intersecting at H is arbitrary; but if they cut each other acutely, it is obviously difficult to locate the precise point of intersection.

Not only in the bisection of angles, then, but in various other operations, the draughtsman should always keep in mind the two following important principles, viz.:

30. 1. **Points for locating lines should be as far apart as possible.**

2. **Lines for locating points should cut each other normally, as nearly as possible.**

31. The necessity of exercising great care at every step in constructing these and similar diagrams will be appreciated if we consider for a moment what tangency really means. Supposing the instrument lead to be sharpened, as it should be, to a keen edge, so as to draw a fine clean line, two tangent circles ought to appear as at A, Fig. 11; merging at the point of contact into one line without increased thickness. A very slight error in the radius or the position of the centre of either of the circles will lead to conspicuous faults, shown at B and C. That there should be no increase of thickness at A, is clear from the consideration that the pencil (or ink) marks represent mathematical lines, which have no thickness; so that two, or twenty, are no thicker than one.

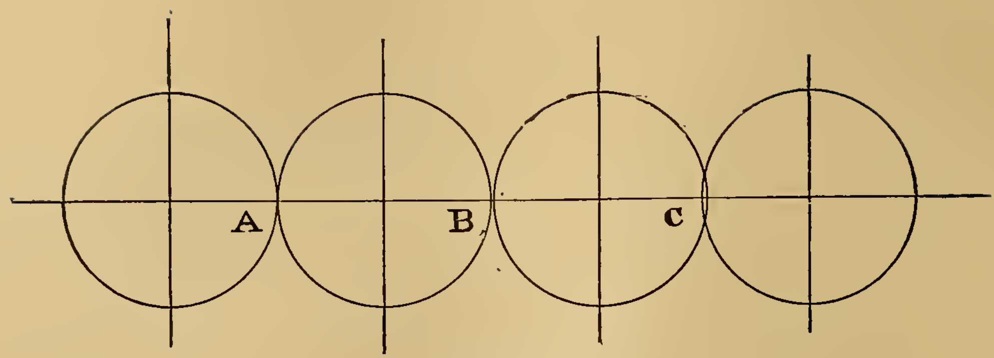

FIG. 11.

32. In inking in, however, a much more effective appearance may be given to such exercises by using lines of different thickness; the centre-lines and construction-lines being very fine, while the given figure and the required tangent-circles are made much thicker. This is entirely distinct from the use of shadow-lines: all the heavy lines here referred to are made of uniform thickness, and perfectly smooth and even. In this way the data and the results, being made bold and prominent, strike the eye at once, thus better exhibiting the nature of the problem; while the processes by which the results are reached, though clearly indicated, take their proper secondary place.

Use of Broad Lines.—It is sometimes necessary to represent a line by a broad band, as for instance in making lecture-room diagrams, which must be distinctly visible to the whole audience. When seen from a sufficient distance, such a band will appear like a fine line, the position of which is mentally referred, not to one or the other edge of the actual stripe, but to its centre. Such diagrams should therefore be drawn at first with at least moderately fine lines, *on each side of which*, at equal distances, parallel lines may be drawn, the space between them being subsequently filled in.

33. It follows from this, that no matter how heavy the lines of the triangle and the circles in Figs. 9 and 10, they will be no thicker at the points of tangency than anywhere else. When viewed from a short distance, the lines no doubt seem to coincide for a sensible extent, instead of at a single point: but so would finer lines if put under a microscope; and if the eye be sufficiently removed, these heavier ones will appear truly tangent, as in fact they are. Beginners are apt to think that two tangent arcs should, when drawn with broad lines, appear as in Fig. 12, *A*, because, as they say, the lines just touch each other. The fallacy will be clear if this method is applied to a reverse curve composed of two tangent

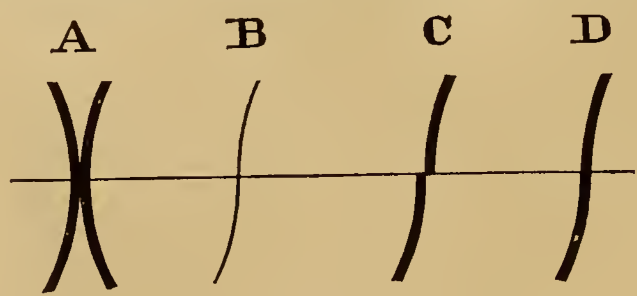

FIG. 12.

arcs, as at *B*; the effect is shown at *C*, one glance at which is enough, it being perfectly obvious that the result desired is that shown at *D*, which is produced as above explained.

34. Dotted Lines.—Some of the lines in Figs. 9 and 10 are what are technically called "dotted;" which strictly means *broken*, that is, formed of alternate short dashes and spaces, for true dots cannot be made with a drawing-pen, nor would they equally well convey the idea of *direction*, if they could be.

In working drawings, dotted lines are used to indicate the outlines of parts concealed by intervening objects; but as to lines used merely in construction, there is no absolute rule for determining which ones should be dotted and which should not. But in all cases the dotting should be regular and even, the spaces just long enough to be distinctly seen, and equal in length to the dashes. If the dashes are long, no matter how regular, the effect is that of apparent haste; and no one minor thing adds so much to the finish of a drawing as dotted lining which is fine, firm, clear, and well spaced.

35. Figs. 13 and 14 are exercises precisely similar in character to those given in Figs. 9 and 10. In Fig. 13 the centres and points of tangency divide the diameter into six equal parts: they may therefore be at once marked at one setting by means of the scale, by selecting a suitable value for *LM*, and making *LR* six times as long.

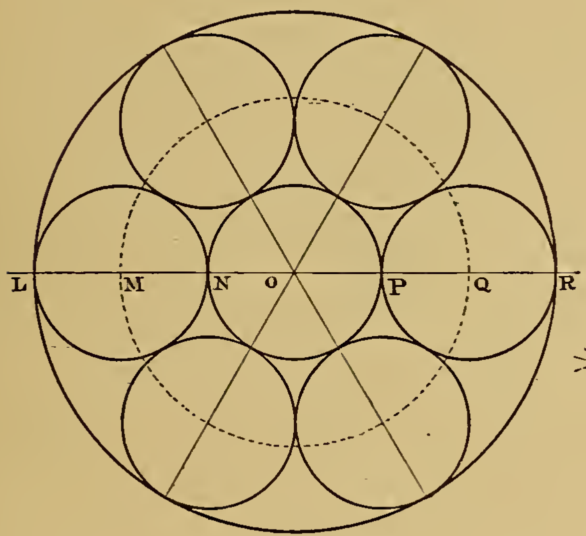

FIG. 13.

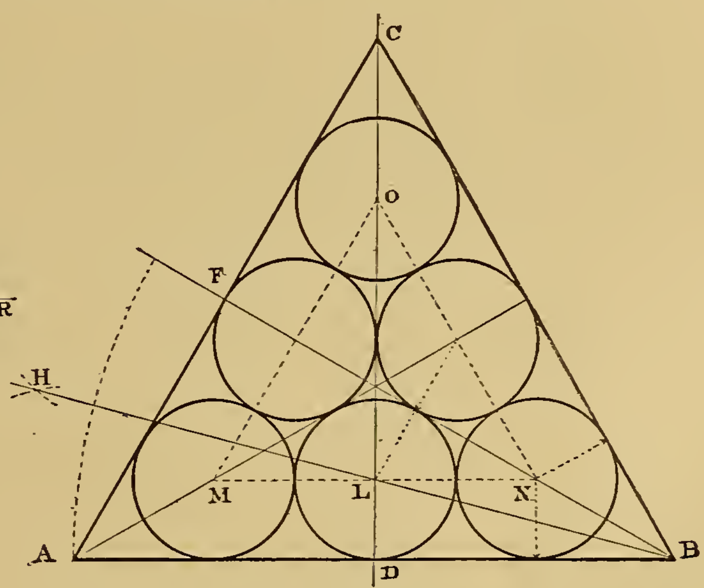

FIG. 14.

In Fig. 14 construct the triangle by means of the scale, as in Fig. 10; bisect the angle *ABF* by the line *BH*, cutting the vertical *CD* in *L*, the centre of one of the required circles; the other centres are determined by drawing the equilateral triangle *MLNO*.

36. Judicious Arrangement of Diagrams and Drawings.—It is understood that, as intimated in (6), the border-line is to be first drawn, and the exercises arranged within the space thus enclosed. In order that the sheet when completed may present a neat and well-balanced appearance, due attention must be paid to the relative positions as well as the actual dimensions of the figures which compose it. The two pairs of diagrams mentioned in the preceding section sufficiently illustrate the fact that perfect symmetry is not always attainable;—which is here emphasized, because experience has shown that beginners are very apt to think that when, as in these cases, two figures are to be drawn, the first thing to do is to draw a vertical line through the centre of the paper. Nothing could be more erroneous,

as the reader will at once perceive by imagining a border to be drawn, say, around Figs. 9 and 10. Were the space within the border thus divided into two compartments or panels, the diagrams would have to be made so small as to be out of all proportion to so large a sheet. Should this dividing line be inked in, the effect would be more tolerable; but if not, the two figures would, by reason of the great distance between them, appear as if they repelled each other, and were crowded out toward the border-line.

The fact is, that one diagram is in a sense a border to the other; and even if, as in some of the following examples, they are perfectly symmetrical, it should be kept in mind that the space between the two need not be any greater than that between either of them and the adjacent end line of the border.

37. Fig. 15 is an open-work eight-pointed star enclosed within two interlacing hollow

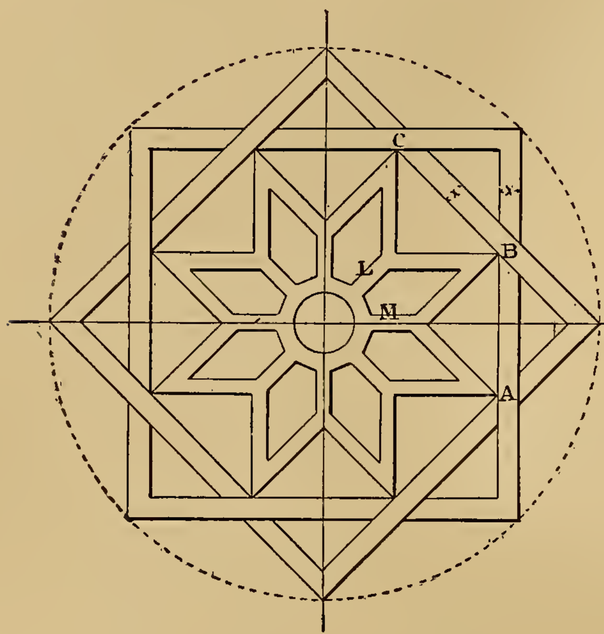

FIG. 15.

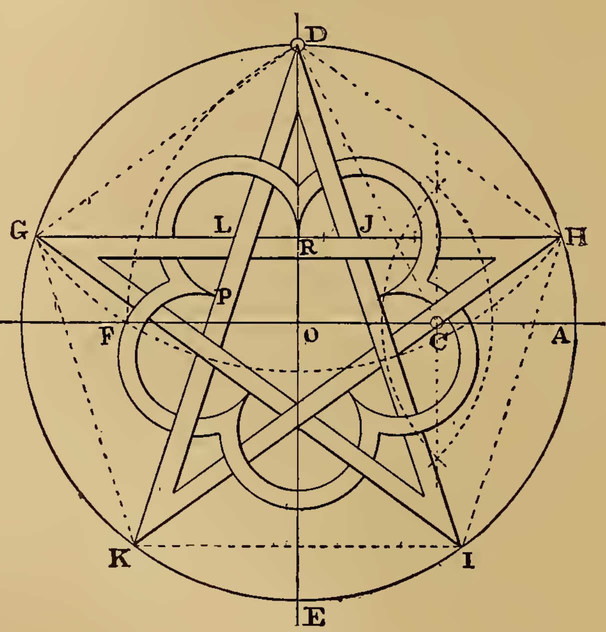

FIG. 16.

squares. The latter are first drawn, and the breadth x evidently determines the positions of the points A, B, C, etc., of the star. The radiating arms L, M, etc., might be laid out by first drawing a centre-line for each, and setting off the half-breadth on each side with the spacing dividers; but a better method is illustrated in Fig. 17. Draw in pencil a small circle at the centre of the star, of a diameter equal to the breadth of the arms, whose outlines are drawn tangent to this circle by means of the T-square and the triangle. The eye can judge with great nicety as to the fact of tangency; and this method will be found of great advantage in many similar cases.

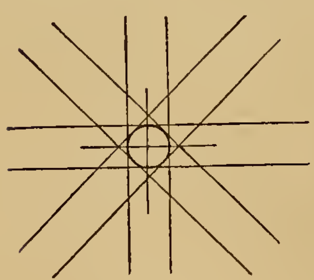

FIG. 17.

38. Fig. 16 is another example of interlaced work, the fundamental figure being a five-pointed star. To locate the points of this star means to inscribe the pentagon in the circle, which is done as follows:

1. Bisect the horizontal radius OA at C.

2. About C describe an arc through D, the extremity of the vertical radius, cutting the horizontal diameter in F. Then $DF =$ side of pentagon.

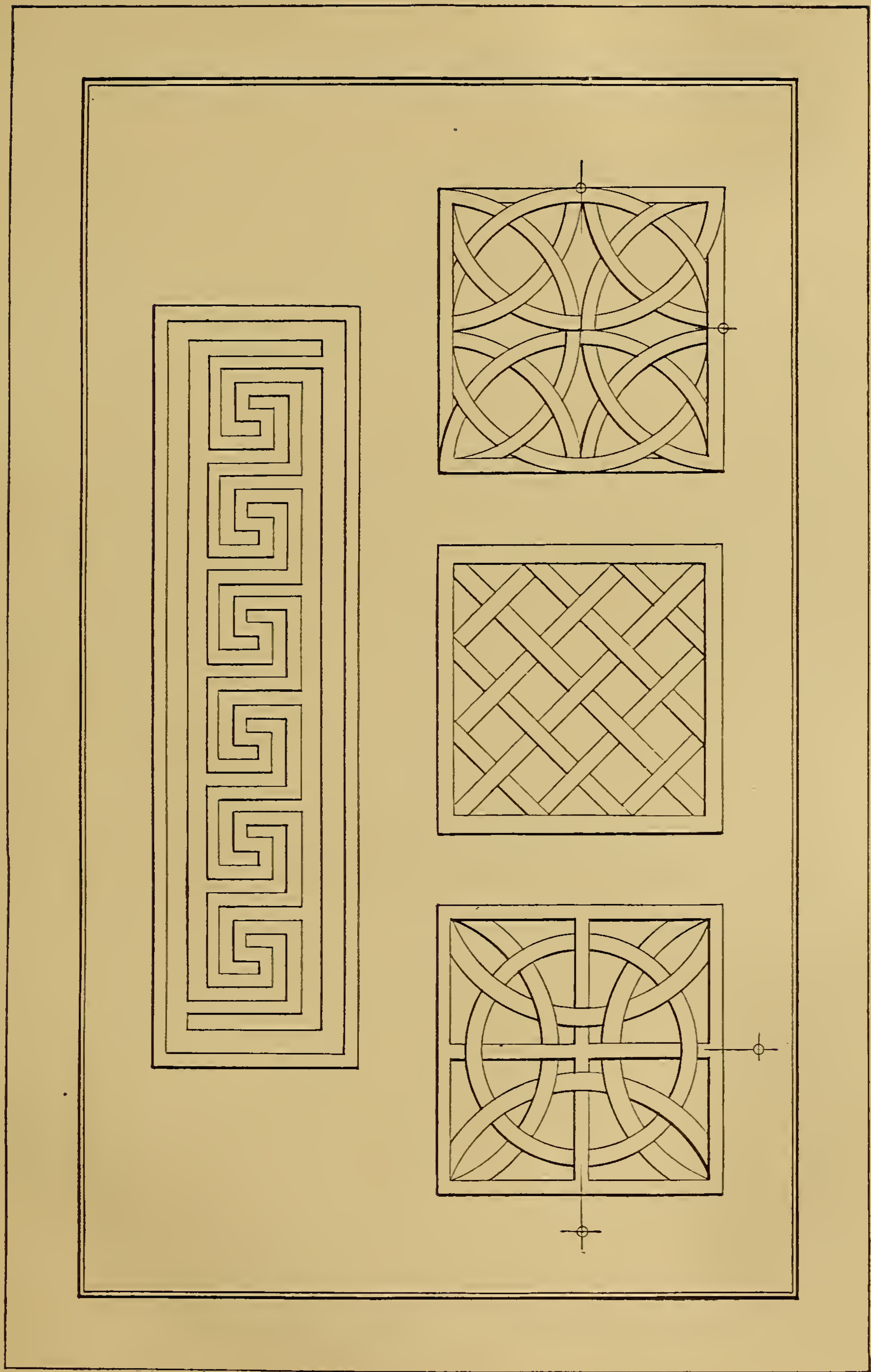

Fig. 18.

The centres of the circular arcs forming the inner interlacing figure are the intersections *J*, *L*, etc., of the outer lines of the star. The radius of all the inside arcs is equal to half *JL*; these arcs are therefore *tangent* to each other at *R*, *P*, etc.; and great care must be taken in manipulating the shadow-lines, in order to produce the *sharp* knife-edge finish essential to a proper effect.

39. In the drawing of interlaced work it is clearly impossible to leave out in pencilling all lines which are not to be inked in, because it cannot be known just which ones they are. Since then all the lines must be pencilled in at first, crossing each other as they may, this should be done very lightly; after which it is advisable, as a precaution against mistakes with the pen, to erase carefully with the sharp corner of a small piece of rubber those portions which are not to be drawn in ink. Work of this description gives good training in the use of the pen, since, in order to avoid a ragged appearance of the drawing, the lines must meet sharply without overrunning; its perfect execution requires both care and skill.

40. Fig. 18 represents a completed sheet, in which the three upper figures are simple exercises in interlaced work; the locations of the centres of the circular arcs being indicated, no explanation is necessary. The lower figure is a panel ornamented with fretwork in the Greek style, the panel being sunken and the fretwork raised; the effect being made pronounced by the use of quite heavy shadow-lines. This pattern, though very simple, is yet of such a nature that the attempt, by using the scale for each member and part of a member, to end *every* line in pencil just where it is to end in ink, would be clearly absurd.

Rule the horizontal lines very lightly from end to end, as indicated by fine lines in Fig. 19;

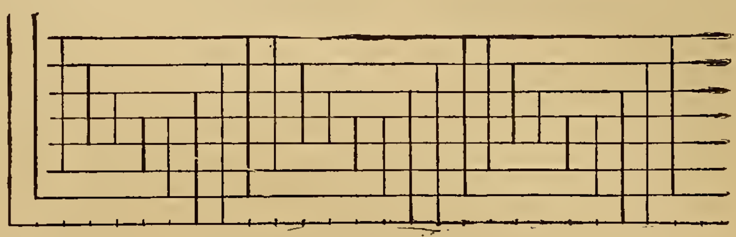

FIG. 19.

lay the scale on the lower line, and with a sharp pencil set off the longitudinal spaces; then by means of the triangle the *vertical* lines can at once be drawn of exactly the right length. After this is done, it will be safer to pencil in a little more strongly, by means of the T-square, those portions of the horizontal lines which are to be drawn in ink; without this precaution mistakes with the pen are likely to occur.

41. Fig. 20 is an exercise which, while apparently simple, is yet a very difficult one to execute perfectly. The diameter of the outer circle being divided into any number of equal parts, begin at one end, and describe a semicircle upon each part, on the same side of the diameter. Then, beginning at the other end, do the like in reverse order on the opposite side of the diameter. Thus at each extremity of the diameter a number of arcs are tangent to each other; in order to prevent blotting, begin with the outer one, and wait until it is dry before drawing the next. This order is also best for another reason—the lines, having some thickness, actually meet each other before reaching the point of tangency; hence the pen need not always start from that point, but may first touch the paper where the lines coalesce.

Moreover, the curvilinear angles become less and less toward the outer part, so that the *daylight* between the arcs should extend farther in toward the point of tangency, as the radii diminish; this requires close attention, and can be better controlled by following the above order in drawing the semicircles.

Instead of assuming at random a radius for the outer circle and using the dividers for subdividing it, thus marring the paper, it is advised to set off an assumed unit the requisite

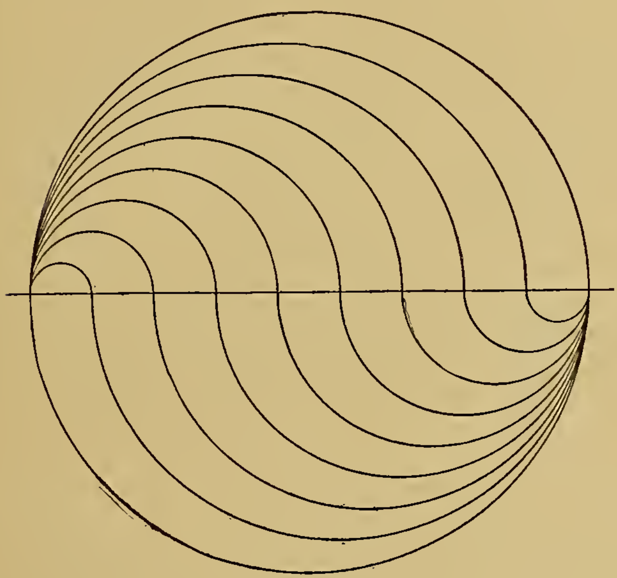

FIG. 20.

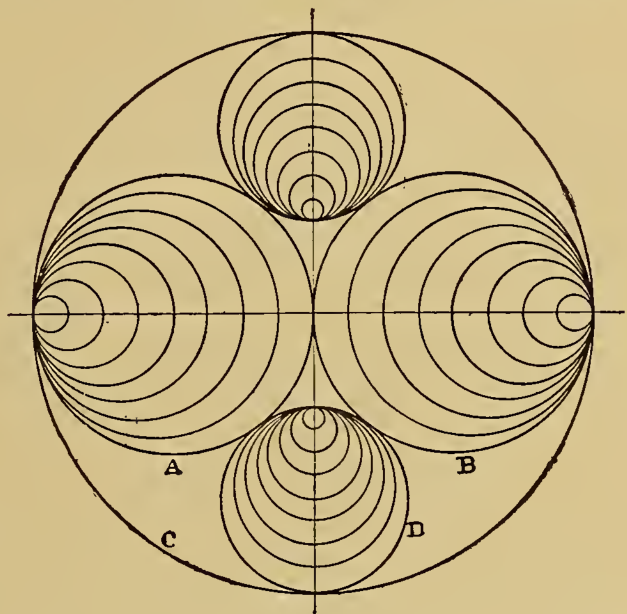

FIG. 21.

number of times by means of the scale and pencil, thus locating at once both the centres and points of contact of the various semicircles.

42. Fig. 21 is an exercise of analogous character, in which the subdivisions on the horizontal diameter may also be most advantageously set off by using a scale, as above explained, the diameter itself of the outer circle *C* being also thereby determined.

This being done, it is clear that the centre of the circle *D*, which must be tangent to the three circles *A*, *B*, and *C*, must lie on the vertical diameter of *C*. The radius of *D*, and the location of its centre, may best be found by trial and error: this will give a result practically as accurate as that of a geometrical construction, which may be made, but the process is long and tedious. The diameter of *D* must then be subdivided, by the scale if practicable, and if not by means of the dividers, in order to locate the centres of the remaining circles.

43. In Fig. 22 the horizontal and vertical diameters may be drawn with the T-square, and the inclined ones with the 45° triangle; but the work should afterward be carefully tested with the dividers, to make sure that the outer circle is accurately divided.

Produce *IK* to cut at *B* the horizontal tangent through *A*, making *AB* equal to *AO*. Then a line bisecting the angle *ABO* will cut *AO* in *C*, the centre of one of the larger circles required. Draw through *C* a horizontal line cutting *OK* in *E*: then *CE* = *CO*, and *E* is the centre of one of the smaller circles, each of which is tangent to the two adjacent larger ones as well as to the exterior circle.

44. The equilateral Gothic arch, Fig. 23, is drawn by describing about *A* and *B*, the extremities of the base, two circular arcs with radius *AB*, which intersect at *C* on the vertical

centre-line *CD*. The problem is to inscribe three tangent circles in this arc; the two lower ones will evidently be equal, and the centres may be found as follows: Set off on the prolongation of *BA* a distance measuring 3 on any scale of equal parts; at *E* set up a

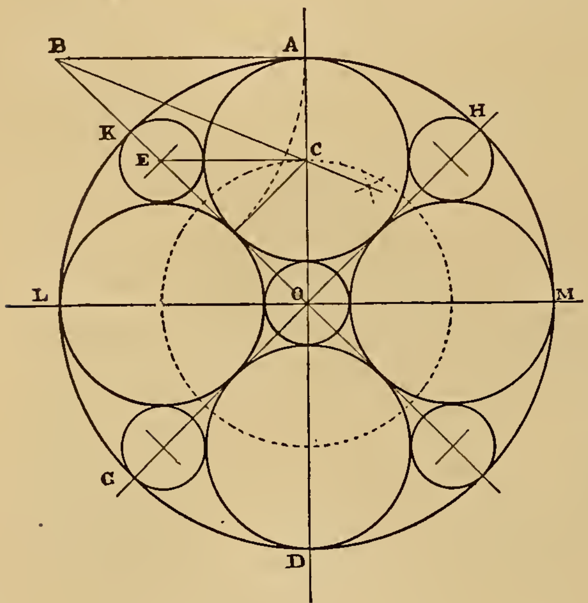

Fig. 22.

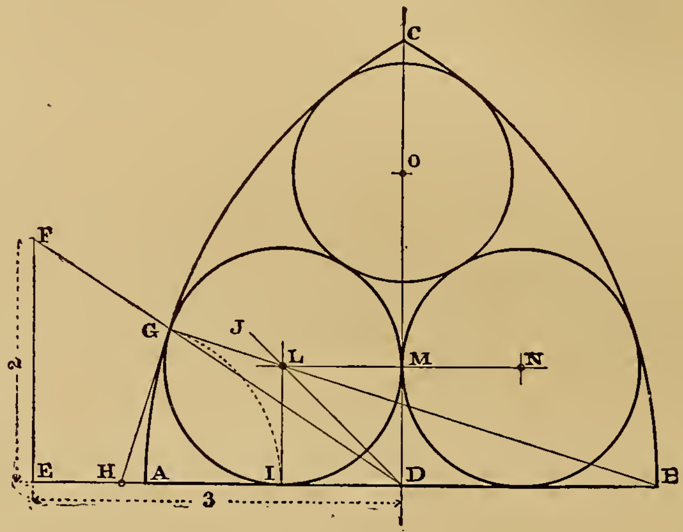

Fig. 23.

vertical *EF*, measuring 2 on the same scale, and draw *FD* cutting the arc *AC* at *G*. Draw *GB*, and also a line perpendicular to it at *G* (which is therefore tangent to the arc *AC*), and cutting *DE* at *H*. On *ED* set off $HI = HG$, and at *I* erect a vertical line cutting *GB* at *L*, the centre of one of the required circles. As a test, bisect the angle *ADC* by the line *DJ*, which should also pass through *L*. Draw through *L* a horizontal line cutting *CD* at *M*, and on it set off $MN = ML$: then *N* is the centre of the second lower circle. The radius of the third circle will be less than that of the others: its centre will obviously lie upon *CD*, and both centre and radius are practically best found by trial and error.

45. Fig. 24 is a panel ornamented with interlaced work, in which the pattern is composed wholly of circular arcs. The radii are uniform throughout, and it is apparent that all the

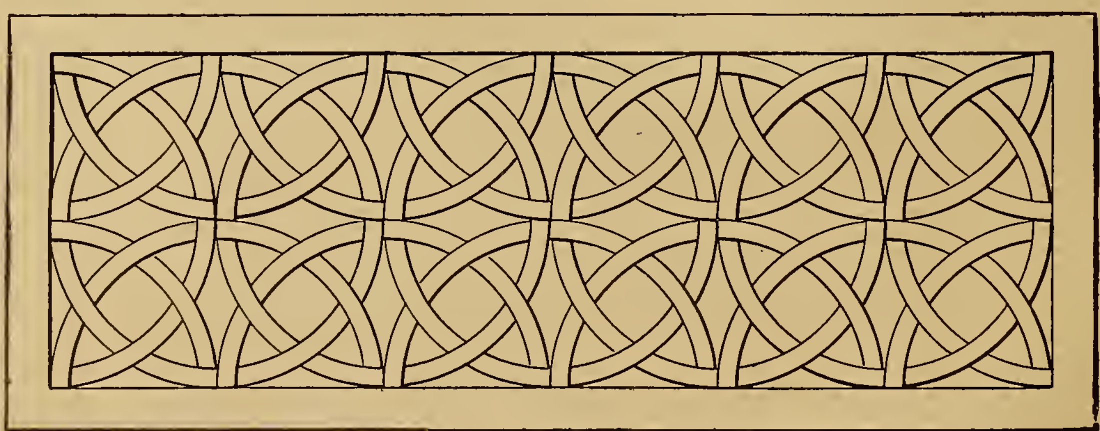

Fig. 24.

centres may be readily set out by using the scale alone as before described. This exercise is an excellent one for acquiring control of the compasses, since in order to produce a good

effect the shadow-lines must be of uniform strength as well as nicely graduated, and as in all work of this description the lines must meet without overrunning.

46. Figs. 25 and 26 are further exercises in "confined tangencies." In Fig. 25, having drawn the circle *O* and the trisecting centre-lines, we may draw the circles *A*, *B*, and *C*, with

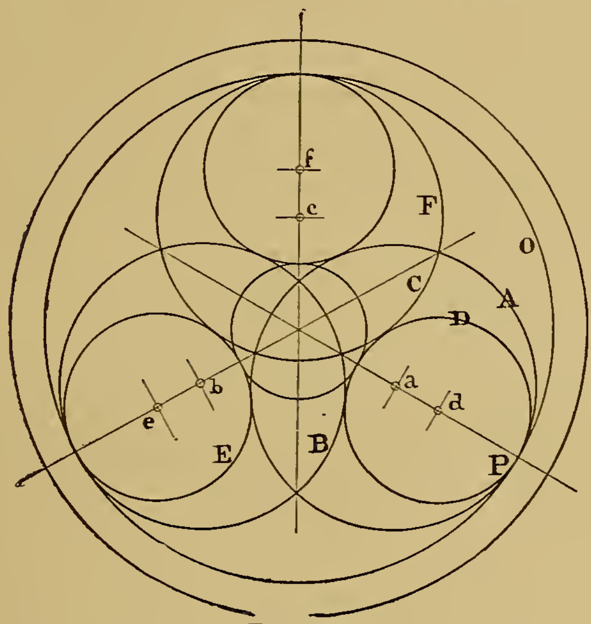

FIG. 25.

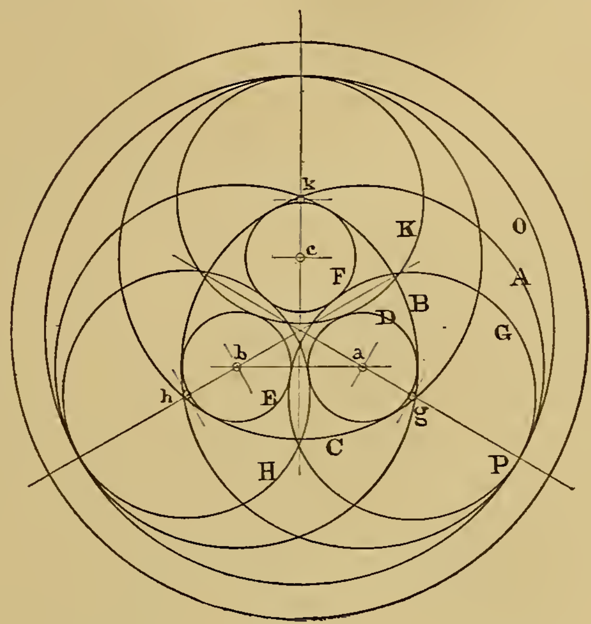

FIG. 26.

any assumed radius as *aP*. Then the circle *D* must touch the circle *O* at *P*, and also be tangent to *B* and *C*; its centre, which must lie on *aP*, as well as its radius, may best be found by trial. Fig. 26 is very similar in its nature; but in this case the radius of *A* is so large that

FIG. 27.

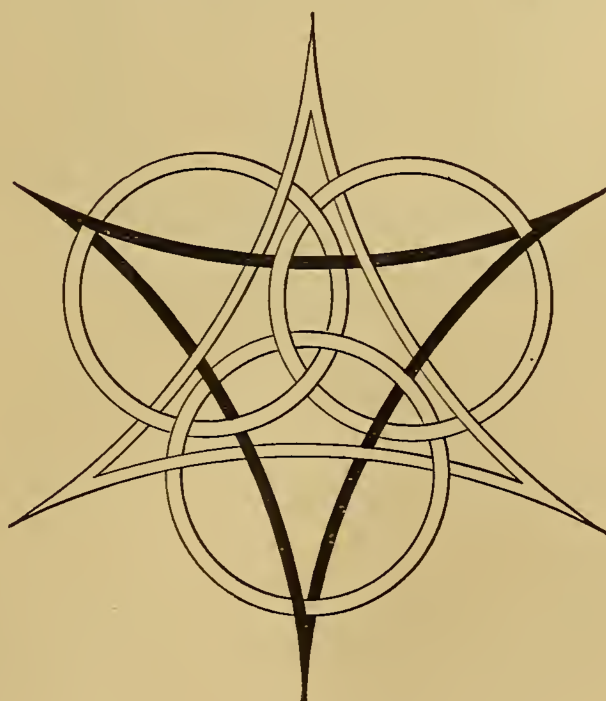

FIG. 28.

its centre *a* lies within both *B* and *C*. Another circle may therefore be drawn about *a*, internally tangent to *B* and *C*, whose radius may be found by drawing *ab* and producing it

to cut the circumference of *A*. Finally, by trial we find on *aP* a centre *g*, and a radius such that the circle *G* is tangent to *O* at the point *P*, and also touches the circles *E* and *F*, which are equal to *D*, and concentric with *B* and *C*.

47. Figs. 27 and 28 are merely fanciful designs of interlaced work, in which the centre-lines, which are necessary of course in laying out the work, are not shown. In regard to the former, it is sufficient to say that the centres of the three circles are at the vertices of the inner equilateral triangle, and the compasses must therefore be handled very lightly in order to avoid making unsightly holes in the paper and thus marring the finish. In Fig. 28 the vertices of the interlacing curvilinear triangles must of course lie at six equidistant points upon the circumference of a circle, and the centres of the arcs which form their sides lie upon the prolongations of the diameters passing through those points. The radii of these arcs is arbitrary: indeed, much in relation to the proportions and details of these exercises is purposely left to individual discretion, because training in judgment and forethought is quite as essential to success as that in manual dexterity. In both these exercises the broad black lines are to be drawn as explained in (**20**); and in Fig. 28 special care in inking in is necessary as to the six outer points, particularly in manipulating the shadow-lines upon these acutely intersecting arcs, which should not appear blunted, but terminate in knife-edges, sharp, clean, and well defined.

48. In Fig. 29, having described the outer and the inner circles, whose diameters are arbitrary, about the common centre *O*, divide the circumference of the latter into any number of equal parts at *A*, *B*, *C*, etc., and draw tangents to it at the points of subdivision.

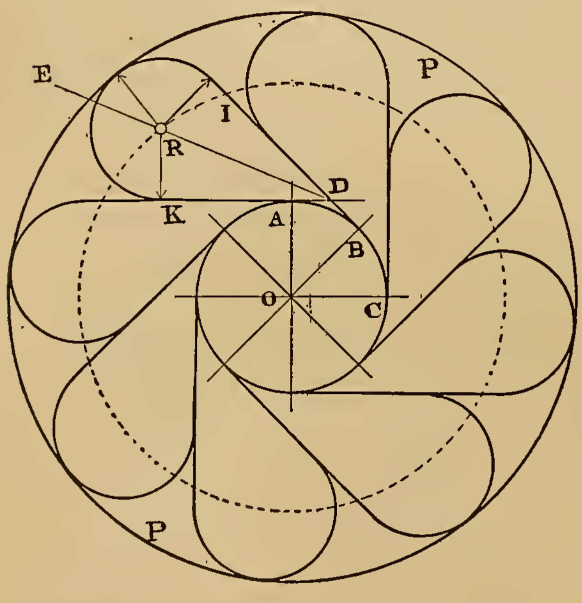

FIG. 29.

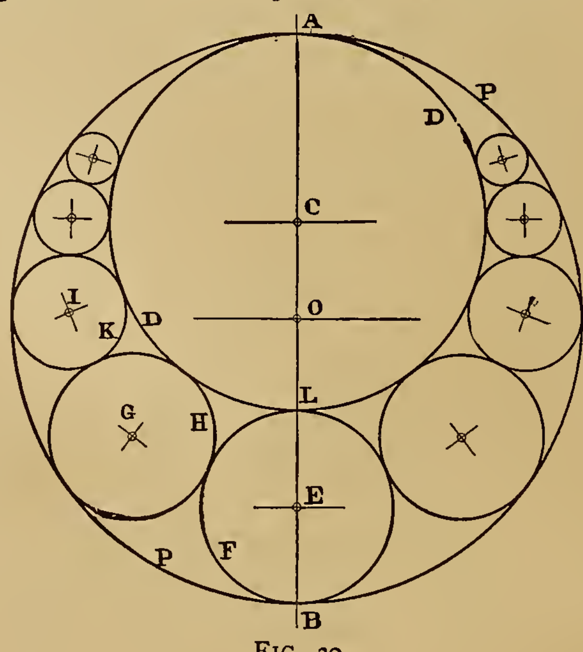

FIG. 30.

The tangent *AK*, produced to the right, intersects at *D* the tangent *BI*; bisect the angle *KDI* by the right line *DE*, on which find by trial the centre, *R*, of a circle which shall be tangent to the outer circle *P*, and also to the right lines *AK*, *BI*. A circle described about *O* through *R* contains the centres of all the small circles similarly tangent to *P*, which of course are equidistant from each other.

In Fig. 30, having drawn the outer circle *P*, and the vertical centre-line *AOB*, describe the

circle D tangent to P, with any radius CA taken at pleasure; bisect LB at E, which is the centre of circle F. Then find by trial the centre G, and the radius, of a circle H, tangent to P, D, and F; also of a circle K, tangent to P, D, and H, and so on as far as may be desired. It is easy to show that the centres of all circles tangent to P and D lie upon an ellipse, of which O and C are the foci, and AE is the major axis; but even the drawing of this ellipse would be of no practical advantage.

49. Outlines composed of Circular Arcs.—In drawings of mechanical subjects it is frequently desirable to make up curved outlines of a series of circular arcs of different radii, in order that the workman may readily copy them with the tools which are always at hand. This will in many cases give an approximation, sufficiently close for practical purposes, to a form which it would require much more time and trouble to lay out with perfect accuracy.

In Fig. 31, $EFGH$ is a rectangle divided by AC into two equal squares. In each square draw the diagonals, intersecting at B and D. About A as a centre describe the arc EF; about B describe the arc FG, then about C and D describe the arcs GH, HE; thus forming

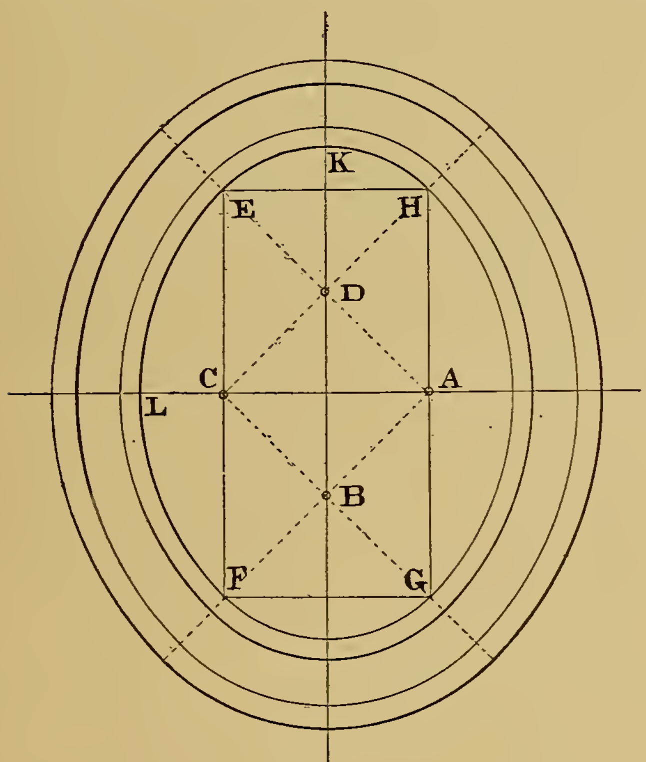

FIG. 31.

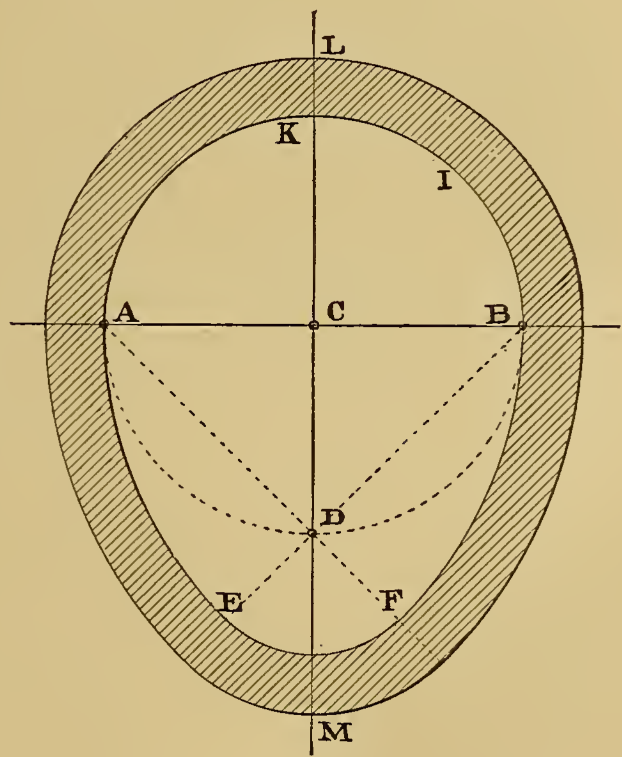

FIG. 32.

a closed figure, approximating in form to an ellipse. Using the same centres, with radii increased by a certain amount, this figure is surrounded by a narrow band of uniform width. Repeating the same operations with still greater radii, but *keeping the same centres*, the result is a representation of a picture frame with two raised mouldings.

It is clearly necessary that these various arcs should be truly tangent to each other, and the junctions imperceptible, as though each outline had been drawn by one continuous motion of the pen.

Relief is given by the use of shadow-lines: in regard to which it may be said that practically the portion LEK may be made of uniform thickness, the tapering of the line being

confined to the half-quadrants LF, KH; and the same holds true in relation to the other shadow-lines upon the mouldings.

50. Fig. 32 shows an approximation to an *oval* outline by means of circular arcs. Describe about C a circle cutting the horizontal diameter at A and B and the vertical diameter at D; draw AD, BD, and produce them. About A as a centre, with radius AB, describe the arc BF; about B with the same radius describe the arc AE, and complete the inner oval by drawing the arc EF about centre D. The outer oval is drawn by means of arcs *about the same centres*, but with radii increased by a definite amount assumed at pleasure. The space between the two ovals may then represent the outer wall of a sewer, and "sectioned" accordingly, the effect being heightened by the use of shadow-lines. As in the preceding example, the shadow-line may be made uniform from A to K, and tapered only from K to I, and from A to E: and similarly with regard to the exterior outline.

TENTATIVE AND MECHANICAL PROCESSES LEGITIMATE.

51. Before going farther, a little more may be said about the methods employed, to which allusion was made in (**3**). In some of the preceding explanations determinations by "trial and error" instead of by geometrical construction have been advised, which may to some appear at first glance inconsistent.

But in drawing we have to do only with the *representations* of magnitudes: thus a mathematical line, which has no actual thickness, must be represented by a mark of perceptible breadth. And the practical limit of accuracy is measured by the actual breadth of the finest visible line. Again, the ultimate test of the precision with which a geometrical operation has been executed lies in the answer to this question: Are the results which should follow actually obtained? To illustrate: Let it be required to draw a circle tangent to three given circles. The centre and radius may be determined by a geometrical construction; but if, when this has been made, the required circle sensibly fails to touch the given ones, it follows that an error has been committed. On the other hand, if by trial a circle be drawn which does touch the others, that fact proves that the centre and radius are those which *should* have been found by the construction; they may therefore be accepted as correct.

52. No positive rule can be laid down for determining which method should be adopted; but if the geometrical operations are complicated, or the conditions such as to make any of the steps dubious, owing to acute intersections or like contingencies, it is easy to see that their results may be less reliable than those of the tentative processes above suggested.

The use of the latter is evidently legitimate in laying out working drawings and the like, since in general the result only is to be represented. But even in making a diagram for the express purpose of illustrating a process, it is perfectly fair to make sure of the result by such means first, and to introduce the proper construction-lines afterward. Again, the draughtsman has often to perform operations for which there *is* no geometrical construction, as, for instance, in drawing a wheel with thirty one teeth: he must divide the circumference of his circle into the given number of equal parts, and this can be done only by trial. Even if the subdivision can be made geometrically, as in constructing the pentagon, Fig. 16, the final test is by trial, to see that the chord is correct; and so, in general, the one method may be used as a check upon the other.

53. Closely akin to these are certain mechanical methods constantly employed by every expert draughtsman. It need hardly be said that he who is possessed of two trustworthy triangles must also be possessed of an evil spirit if he does not use them for drawing lines parallel or perpendicular to each other, instead of resorting to a geometrical construction. This is no more mechanical than laying off an angle by means of a protractor: in either case the precision of the work depends upon the perfection of the instruments and the skill with which they are used, and indeed the same may be said of drawing a straight line through two given points by means of a ruler. In doing this the edge of the ruler is set, not so as to coincide with either point, but at a small distance from each, the eye being able to judge with perfect certainty of the equality of these distances, or as it is technically expressed, of the "daylight," between the ruler and the points. And this judgment is equally reliable if one or both the points be replaced by a curve. In this way it will be seen that if it is required to draw a tangent to a given curve through a given point or in a given direction, or a tangent to two given curves, the operations may be performed with the ruler alone with as much precision as though the points of contact were determined by geometrical processes.

54. It may be accepted, then, that the eye can judge of the *fact* of tangency between a curve and a right line when the direction of the tangent is either assigned or fixed by some other condition, with a degree of accuracy equal to that attainable in any graphic operation whatever. And no argument is needed to show that it would be absurd not to take advantage, in drawing mechanically, of a fact which in numberless cases enables the draughtsman to save both time and trouble, without in the slightest degree affecting the reliability of the results. But it is to be understood that the *point* of tangency cannot be located by the eye with any precision, and must in all cases be determined by some process of construction.

CHAPTER II.

EXERCISES IN THE DRAWING OF NON-CIRCULAR CURVES.

55. It is often necessary, particularly in drawings of machinery, to construct with accuracy various mathematical curves; and some preliminary practice in this kind of work being desirable, the following chapter is devoted to explanation of some of the more important of these curves, and of various graphic processes relating to them. In these and other operations there is frequent occasion to rectify circular arcs, and to lay off, either upon circles of given radii, or so as to subtend given angles, arcs of given linear values.

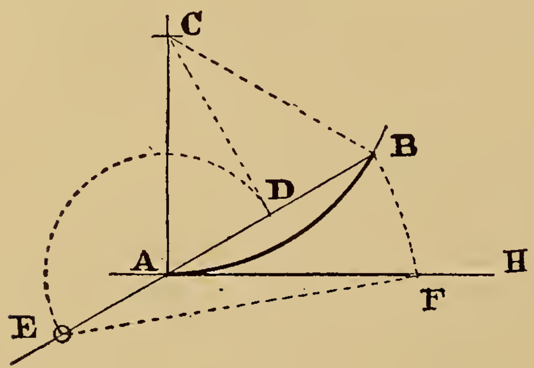

FIG. 33.

In such cases the following constructions, due to Prof. Rankine, are extremely convenient; the results are obtained much more expeditiously than by computation, with a degree of accuracy amply sufficient for all ordinary purposes.

56. I. *To lay off on a right line a distance approximately equal in length to a given circular arc.*

Let AB, Fig. 33, be the given arc, and AH its tangent at A. Draw the chord BA, and produce it; bisect AB in D, and set off AE equal to AD. About E as a centre, with EB as radius, describe an arc cutting AH in F: then $AF =$ arc AB, very nearly.

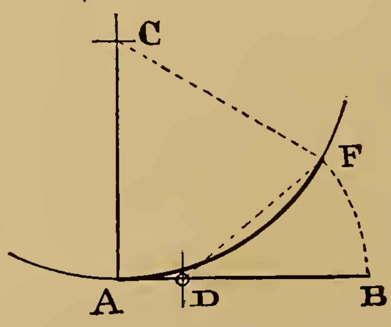

FIG. 34.

II. *To lay off on a given circle an arc approximately equal in length to a given right line.*

In Fig. 34, let AB, the given line, be tangent at A to the given circle. Set off $AD = \frac{1}{4}AB$, and about D as a centre, with radius $DB = \frac{3}{4}AB$, describe an arc cutting the given circle in F. Then arc $AF = AB$, very nearly.

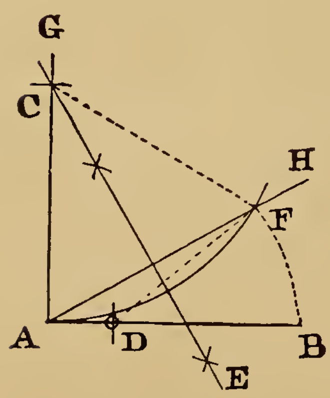

FIG. 35.

III. *To find the radius of an arc which shall subtend a given angle, and be approximately equal in length to a given right line.*

Let AB, Fig. 35, be the given right line. Draw AG perpendicular to AB; and also AH, making the angle BAH equal to half the given angle. Set off $AD = \frac{1}{4}AB$, and about centre D, with radius $DB = \frac{3}{4}AB$, describe an arc cutting AH in F. Bisect AF by the perpendicular EC, which will cut AG in C, the centre of the required arc AF.

57. Amount of the Error in the above Processes.—According to Prof. Rankine, the straight line is a little less than the arc in the application of either of these rules. He gives

the magnitude of the error as about $\frac{1}{900}$ part of an arc of 60°; but it varies as the fourth power of the subtended angle, and may be reduced to a very minute amount by subdivision. Thus, for an arc of 30°, the error will be $\frac{1}{900} \times \frac{1}{16} = \frac{1}{14400}$; and for one of 20°, only $\frac{1}{900} \times \frac{1}{81} = \frac{1}{72900}$. If, then, the arc, given or required, subtends an angle of over 60°, subdivision should be resorted to in practically applying either process.

The first two rules can also be applied to other curves than circles, provided that the change of curvature in the part to be dealt with be small and gradual.

THE ELLIPSE.

58. First Method.—About the centre C, Fig. 36, describe two circles whose diameters are respectively equal to the major and minor axes, AB and MN. Draw any radius at pleasure, cutting the inner circle in D and the outer one in E. Draw through D a parallel to AB, and through E a parallel to MN: the intersection O of these lines will be a point upon the ellipse. This is the most accurate of all methods for constructing this curve by points, all the intersections, D, E, O, being right angles.

To draw the tangent at a given point upon the curve.—Let P be the point: then by reversing the above process we find that P would have been determined by the radius CGH. At G and H draw tangents to the two circles, cutting the corresponding axes produced, in L and R, respectively: then LR is the required tangent at P.

Conversely, to find the point of tangency.—Suppose it required to draw a tangent through a given point Q, or parallel to a given line XY. The tangent is drawn mechanically as explained in (**53**): it cuts the axes at R and L. From R draw a tangent to the outer circle, from L a tangent to the inner circle; these should be parallel to each other, and the points of contact are found by drawing CGH perpendicular to both: then through G draw a parallel to AB, and through H a parallel to MN, which two lines will intersect in the required point of tangency P on the ellipse.

59. Cautionary Remarks.—Particular attention is called to these points, viz.:

1. The radii, CE, etc., need not be equidistant, and no attempt to subdivide the circle should be made.

2. In construction, no radius should be drawn at all; for instance, set the triangle as if to draw CK, but mark only the points I, K: then very short lines only, intersecting at T, need be drawn. See (**4**) page 2.

3. The points determined should be nearer together where the curvature of the ellipse is most rapid; but the actual number is arbitrary. This applies to the construction of *any* curve by locating points.

4. For illustrating the process, two radii are sufficient: these should be made more prominent, as in the figure, than the perpendiculars and horizontals leading to the intersections upon the ellipse, which may be indicated by short full lines.

60. Second Method.—Let C, Fig. 37, be the centre, AB the major axis, DE the minor axis. About D with radius AC describe an arc cutting AB in the foci F, F'. About F as a centre describe an indefinite arc with any radius FH, greater than AF and less than BF. About F' describe another arc, with a radius $F'O = AB - FH$: this arc will cut the one first drawn in O, O', two points of the required curve.

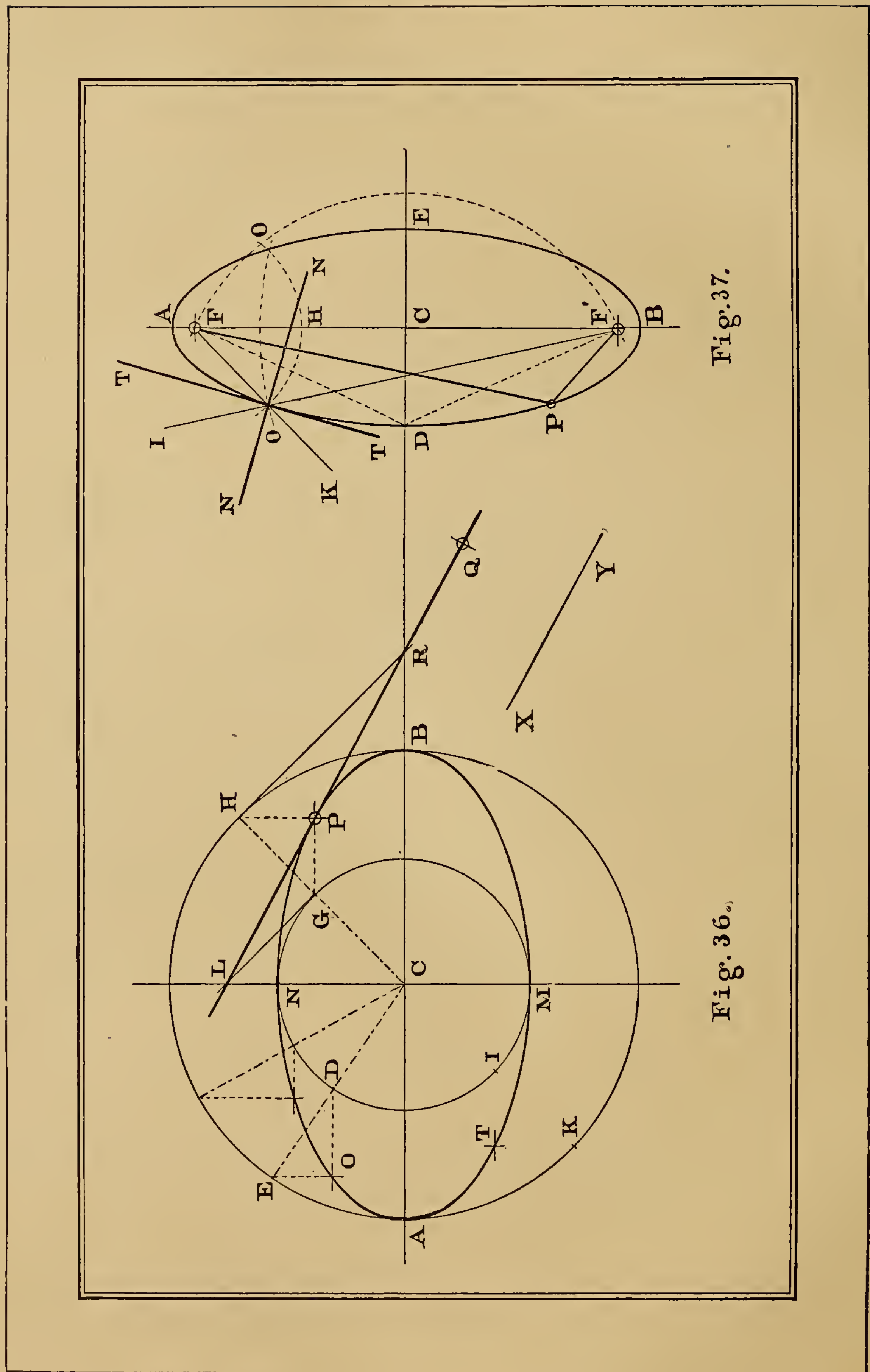

Fig. 36.

Fig. 37.

This method is not eligible for the ordinary purposes of the draughtsman, because it involves much more labor than the preceding one, and the intersections of the arcs are in many cases too acute to be reliable. But it is of interest as depending upon the property of the ellipse that the sum of the focal distances is the same for every point upon the curve, and equal to the major axis; thus,

$$PF + PF' = OF + OF' = AF + AF' = BF + BF' = AB.$$

61. **Third Method.**—Upon this property, also, depends the operation of drawing the "gardener's ellipse" by the aid of a string and two pins. Let two fine pins be fixed in the drawing-board at F, F', Fig. 37; around these pass a loop of waxed sewing-silk, of which the total length is $AB + FF'$: if this loop be kept constantly taut by a pencil P, the latter in moving will trace the ellipse.

To draw a tangent at any point, as O: Produce FO and $F'O$, and bisect the exterior angles FOI, $F'OK$ by the line TT. To draw the normal at the same point, bisect the angles IOK, FOF' by the line NN. Obviously the axes cut the curve normally at their extremities A, B, D, E.

62. **Fourth Method.**—Fig. 38 illustrates the principle of a common elliptographic

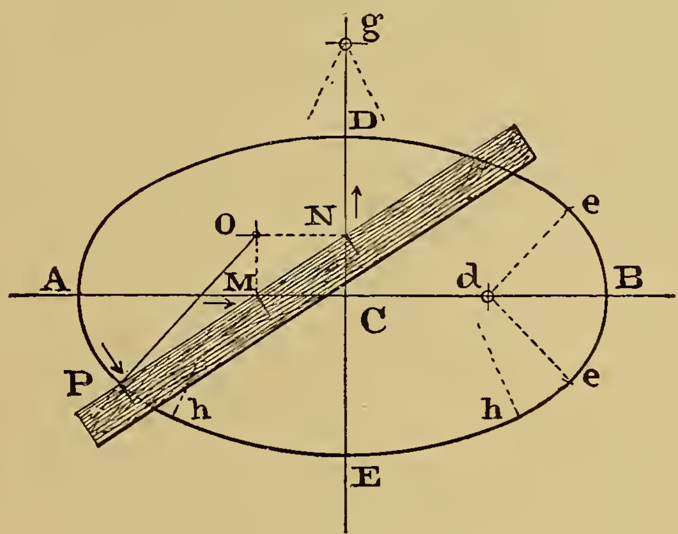

FIG. 38.

trammel. The three points P, M, N are in one right line, the distance PM being equal to CD, and PN equal to CA. Keeping M always upon the line of the major axis, and N upon the line of the minor axis, the point P will at all times lie upon the ellipse.

This method is extremely convenient when the greatest precision is not required; the three points being selected upon the graduated edge of a scale, or marked upon the edge of a smoothly cut strip of paper.

To draw the normal at P. At M draw a perpendicular to AB, at N a perpendicular to DE: these perpendiculars intersect at O, and OP is the normal required; and this is independent of the position of the point P.

63. **Fifth Method.**—To inscribe an ellipse in any given rectangle, Fig. 39. Join the middle points of the opposite sides by the right lines AB, EF: these will be the axes, and intersect in the centre C.

Divide the semi-minor axis CF, and the half side GF of the rectangle, into the same number of equal (or proportional) parts. Through the points of subdivision on CF draw right

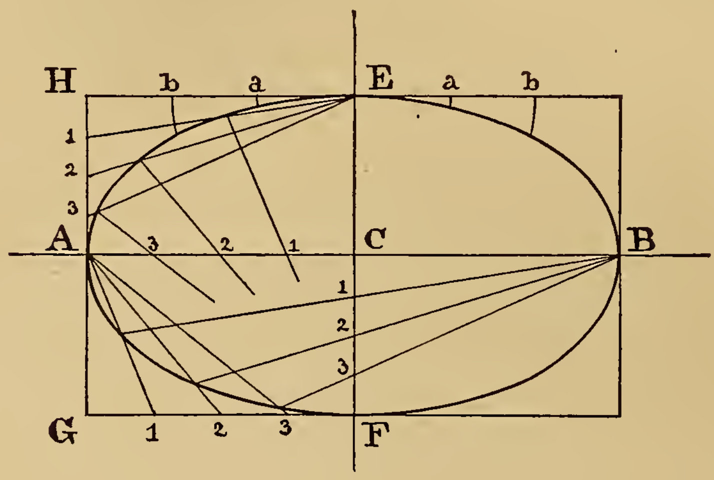

FIG. 39.

lines from B, and produce them to intersect the lines drawn from A to the corresponding points on GF. These intersections will lie upon the ellipse.

Or, divide the semi-major axis AC and the half side AH in like proportion, and proceed in a similar manner, the two series of intersecting lines being drawn from E and F, the extremities of the minor axis.

64. The same process is applicable when it is required to inscribe an ellipse in any given parallelogram, as shown in Fig. 40; but in this case AB, EF are not the axes.

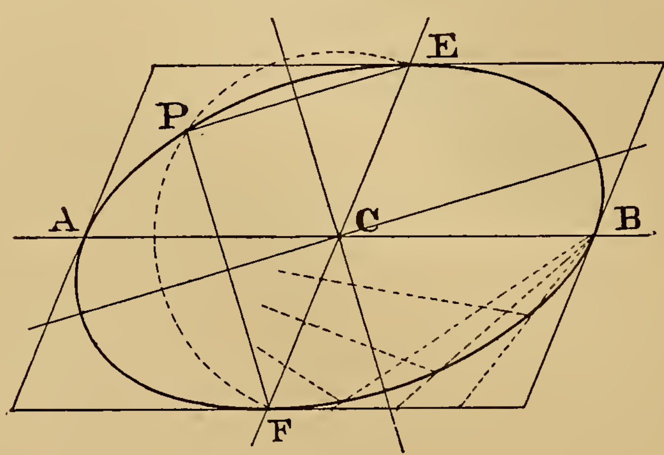

FIG. 40.

They are, however, conjugate diameters, for each is parallel to the tangents at the extremities of the other; and since the parallelogram can always be constructed if AB and EF are given, we have thus a simple and ready method of constructing the ellipse upon any pair of conjugate diameters.

In order to determine the direction of the axes, describe about the centre C a circle upon either of the conjugate diameters, as EF: the circumference cuts the ellipse in P, and the supplementary chords PE, PF are parallel to the axes.

65. Sixth Method.—To construct the ellipse by means of ordinates of the circle. In Fig. 41, let it be required to draw the ellipse of which *CP* is the semi-major and *CR* the semi-minor axis.

Describe a circle with radius $AE = CR$, and divide *CP* and *OA* into any number of proportional parts. At each point of subdivision erect a perpendicular to *CP*, equal to the

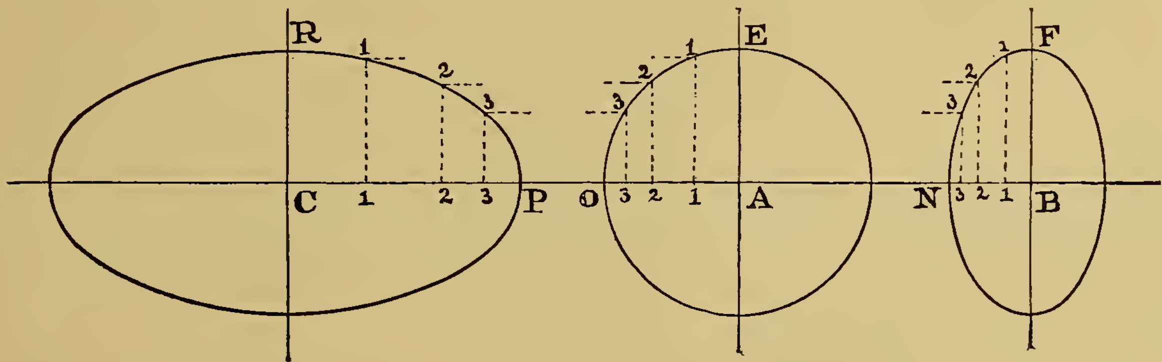

FIG. 41.

ordinate of the circle at the corresponding point of *OA* : the curve *PR* thus determined is the required ellipse.

Or, *AE* may be made equal to the semi-major axis, as *BF*: then if *OA* and the semi-minor axis *BN* be similarly subdivided, the ordinates of the circle will be equal to the corresponding ordinates of the ellipse *NF*.

66. In a given ellipse, to find the conjugate to a given diameter.

In Fig. 42, let *PO* be the given diameter. Draw any chord *EF* parallel to *PO*, and bisect it: then the required conjugate diameter *MN* passes through the point of bisection, and *TT*, the tangent at *P*, is parallel to *MN*.

Otherwise: Draw the chord *EF* parallel to *PO*, and also the diameter *EG*: then *MN* and *TT* are parallel to the supplementary chord *FG*.

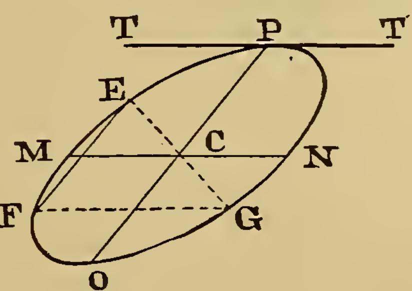

FIG. 42.

67. It is not, of course, intended or expected that an ellipse should be drawn by each of these various methods merely as an exercise: it is proper, however, to illustrate them here, since all these different properties of the curve will be found applicable at one time or another in subsequent operations.

In making the constructions for practice only, it is recommended that two ellipses should be drawn on each sheet, the major axis of one being horizontal, that of the other vertical; that these should be laid out by different methods; and particularly that they should be made of quite different eccentricities: for while the law of the curve is always the same, the actual form is capable of infinite variation, and the course here advised will give greater practice in the use of the sweeps.

PRACTICAL SUGGESTIONS IN REGARD TO THE DRAWING OF CURVES.

68. A sufficient number of points having been determined, the curve should first be sketched in very lightly with the free hand; if there be any serious error in the location of

any points, it is likely to be thus detected. The contour should then be pencilled in more firmly by aid of the "sweeps" or curved rulers, which also are used in finally drawing it with the pen. The pencil line should be fine and clear; if the ink line be made heavy, half its thickness should be outside and half inside of the exact contour: see (32) and (33).

Approximating Circular Arcs.—It is not only legitimate, but advisable, to make use of the compasses in inking in. By trial a radius may be found with which an arc can be drawn, having its centre upon the major axis, which will agree so closely with the ellipse for some distance on each side of the vertex, that the difference is absolutely imperceptible even when the line is as fine as possible. Thus in Fig. 38 the arc *ee* may be drawn about the centre *d*; and in like manner *hh* is an arc described about the centre *g*, on the line of the minor axis. The same radii being used for arcs through the opposite vertices, perfect symmetry is ensured, and there remain only four portions equal to *eh*, to be drawn with the sweeps. The points, *ee*, *hh*, etc., should of course be previously set off, equidistant from the vertices, to secure equality of length in the opposite arcs.

This process is also applicable to many other curves, as the parabolă, hyperbola, sinusoid, etc., which are symmetrical about an axis.

69. In the case of a very long and narrow ellipse, the centre corresponding to *g* in Fig. 38 may become inaccessible; and the part *hh* of the curve must also be drawn by means of the sweeps. It is a very common error to suppose that in any case accuracy can be secured by adjusting a curved ruler to draw from *E* to *h* on one side, and then reversing it to draw from *E* to *h* on the other. Even if the tangent at the vertex be first drawn in pencil, the attempt to do this will almost certainly result in the formation of a "hump" or "broken back" at *E*. The proper course is indicated in Fig. 39. Draw the tangent at *E*: then about *E* as a centre describe arcs cutting both the curve and the tangent on opposite sides of the vertex, as at *a*, *a*, *b*, *b*. The eye can judge accurately of the equality of the corresponding intercepts, and the sweep can thus be adjusted to draw at one stroke of the pen an arc of the curve extending some distance each way from the vertex.

70. In using the sweeps, certain precautions are requisite in order to make a smooth and "fair" line. Beginners are apt to set the sweep so as to draw a line through three or four points, and in readjusting it, to begin at the *last* point of the line just drawn, and set it so as to agree with the next three or four, and so on. The result is likely to be painfully conspicuous by reason of "humps"—the various sections of the curve thus traced not being truly tangent to each other. To avoid this: 1. *Never draw quite as far as you think you can* at one setting of the sweep; in other words, stop before the sweep deviates sensibly from the curve. 2. In readjusting, always set the sweep *so as to coincide* for a little distance with the portion of the curve already drawn.

71. Copying by Transferring or Impression.—The ellipse is composed of four similar and equal arcs, or quadrants. If one of these has been accurately laid out by any of the methods described, the labor of constructing the other three may be saved by the following expedient: Suppose, for instance, that the quadrant *AE*, Fig. 39, has been drawn with precision: upon a piece of thin transparent paper make a *careful* tracing of it with a sharp-edged, moderately hard pencil, and trace also the axes *AC*, *EC*. Turning this paper over, and adjusting it by these centre-lines, go over the visible contour again with a firm pressure:

this will leave a faint trace of the quadrant *EB*, and will also make a mark on the reverse side of the thin paper; which by like treatment will yield an impression of the quadrant *BF*; and *FA* may be also similarly transferred. This simple proceeding can be applied in the case of many other curves composed of similar and symmetrical branches, as well as in the copying of others which are to be reproduced in several different positions. Much labor in the way of construction can thus be saved; but in the various tracings the sweeps must be used as conscientiously and as carefully as though the results were final.

THE PARABOLA.

72. The Parabola, Fig. 43, is a curve every point of which is equally distant from a given point *F*, called the focus, and a given right line *DD*, called the directrix. It is therefore symmetrical about the axis *OFC* perpendicular to *DD*, and its vertex *V* lies at the

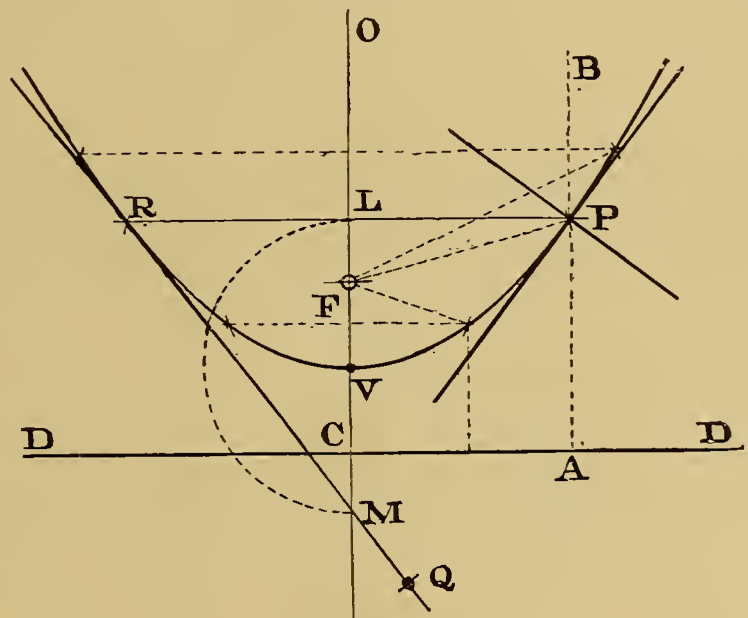

FIG. 43.

middle point of *FC*. Draw any line parallel to *DD*, as *RLP*: then the points in which this line is cut by an arc described about *F*, with a radius equal to *LC*, will lie upon the curve.

To draw a tangent at any point, as P: Draw *FP*, and also *PA* perpendicular to *DD*: then the required tangent bisects the angle *FPA*, and the normal bisects the angle *FPB*.

Otherwise: Let *R* be the given point. Draw *RL* perpendicular to the axis, and on the axis set off $VM = LV$: then *MR* is the required tangent.

To draw a tangent through a point without the curve, and find the point of tangency: Let *Q* be the given point. Draw the tangent mechanically as explained in (53), and produce it to cut the axis in *M*; set off on the axis $VL = VM$, and through *L* draw a parallel to *DD*; this will cut the tangent in the required point *R*. The same process applies to the drawing of a tangent in a given direction.

73. Second Method.—In Fig. 44, let *V* be the vertex, *VO* the axis, and *P* a point through which the curve is to pass. Draw *PR* perpendicular to *VO*, and make $QR = PQ$. On the axis set off $VL = VQ$; draw *PL* and *RL*, divide them into the same number of equal parts, number the points of division in opposite directions, and join the points correspondingly numbered, as 1, 1, 2, 2, etc.: the lines thus drawn will be tangents to the required curve.

To find the point of tangency on any one of these lines, for instance 1, 1. This line cuts the axis at N; set off on the axis, $VK = VN$, and draw KB perpendicular to VO; this will cut 1, 1, in the required point B.

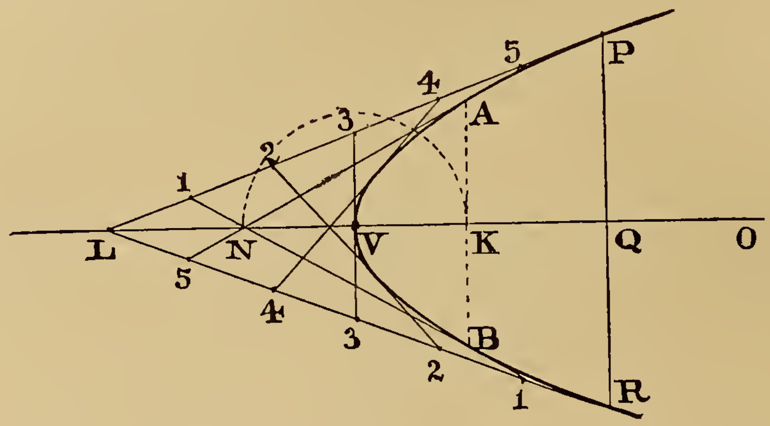

FIG. 44.

74. **Third Method.**—In Fig. 45, let V be the vertex, VO the axis, and P a point through which the curve is to pass. Draw PO, VA, perpendicular to the axis and equal to each other, and join AP. Divide VA and AP into any number of proportional parts; through the points

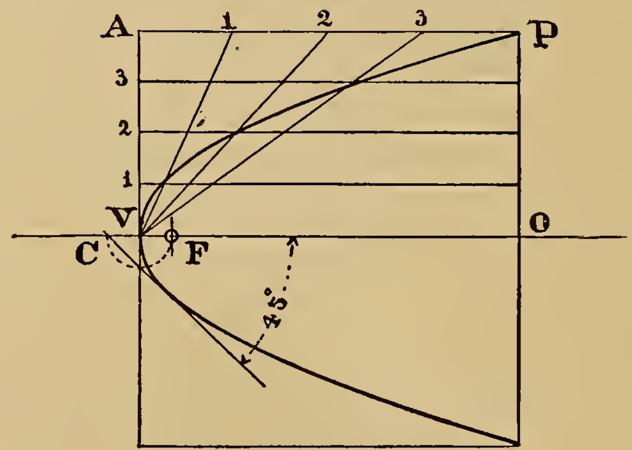

FIG. 45.

of division on VA draw parallels to the axis, and from those on AP draw lines to the vertex: the intersections of lines thus drawn through corresponding points on VA and AP will lie upon the parabola VP.

To find the focus and directrix: Draw a tangent to the curve, making an angle of 45° with VO (72); this cuts the axis at the point C of the directrix DD, which is perpendicular to VO. On the axis set off $VF = VC$: then F is the focus.

75. **Application.**—A pleasing application of this curve to practical purposes is found in the pointed arch, Fig. 46: AC being the span and BD the rise, the outer line of the arch is composed of the two parabolas AB, BC, of which the common axis is AC, and the vertices are A and C. These may perhaps be best constructed by the method of Fig. 45, as indicated by the rectangle $AEBD$, and the two lines intersecting in x.

The inner curves ab, cb are not true parabolas, but *parallels* to AB, BC: that is, the normal distance between the outer and inner curves is everywhere the same. These interior

curves are best drawn thus: Strike with the bow-pencil a series of arcs with a radius equal to the assumed breadth Aa, all the centres being on the parabola AB. Then by means of the sweeps draw the *envelope* of these arcs, that is, a line tangent to them all.

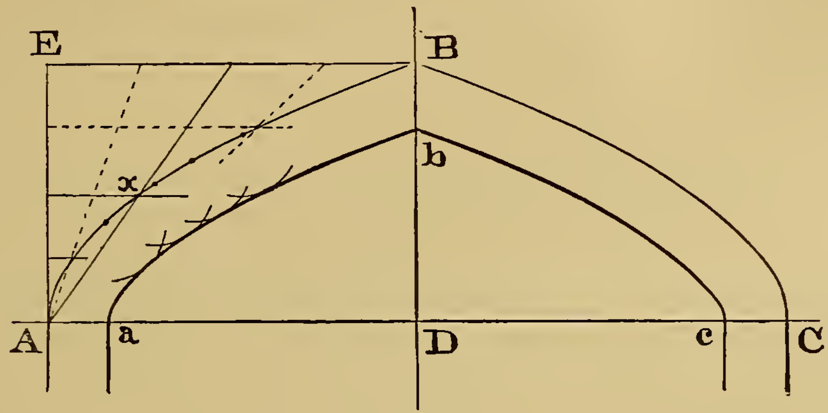

FIG 46.

Note.—Points on this interior curve might be found by drawing normals to the parabola AB by methods previously indicated, and setting off on each the assumed distance Aa. But the process just described is far less laborious, and if the number of arcs be equal to that of the points, it is quite as accurate, if not more so.

THE HYPERBOLA.

76. The Hyperbola is a plane curve generated by the motion of a point subject to the condition that the *difference* of its distances from two fixed points called the foci shall always be equal to a given line, whose length must be less than the distance between the foci.

In Fig. 47 set off on the horizontal line $CF = CF'$, and let F, F' be the foci: also set off

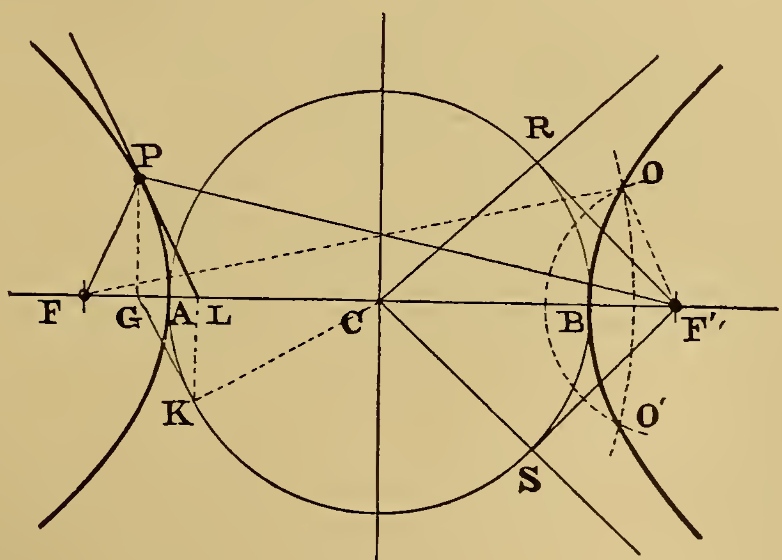

FIG. 47.

$CA = CB$, and let AB be the given constant difference. Clearly, then, $FB - BF' = AF' - AF = AB$; therefore A and B lie on the curve. With any radius FO greater than FB, describe an arc about F as a centre: then about F', with a radius $F'O = FO - AB$,

describe another arc, which will cut the one first drawn in two points O, O', of the hyperbola. Since with the same radii arcs may be described about the other foci, it follows that the curve has two equal and opposite branches; which are infinite, because FO may be of any length.

The point C is called the centre, and the line AB the major axis, to which the minor axis passing through C is perpendicular.

To draw a tangent at any point of the curve, as P: Draw PF, PF', and bisect the angle between them; the bisector is the required tangent. *Otherwise thus:* Describe a circle upon AB as a diameter, let fall a perpendicular PG upon the major axis, and from its foot G draw a tangent to the circle. From K, the point of tangency, let fall a perpendicular KL upon the major axis; through its foot L draw LP, the tangent required. By reversing this process the point of contact may be found, when the tangent is drawn mechanically (**72**), through a given point not upon the curve, or parallel to a given line.

To find the asymptotes: From either focus, as F', draw tangents to the circle described upon diameter AB; find the points of contact R and S: then CR and CS are the asymptotes.

77. Second Method.—In Fig. 48, let C be the centre, AB the major axis, and O a point through which the hyperbola is to pass. Describe upon AB the semicircle ADB, and

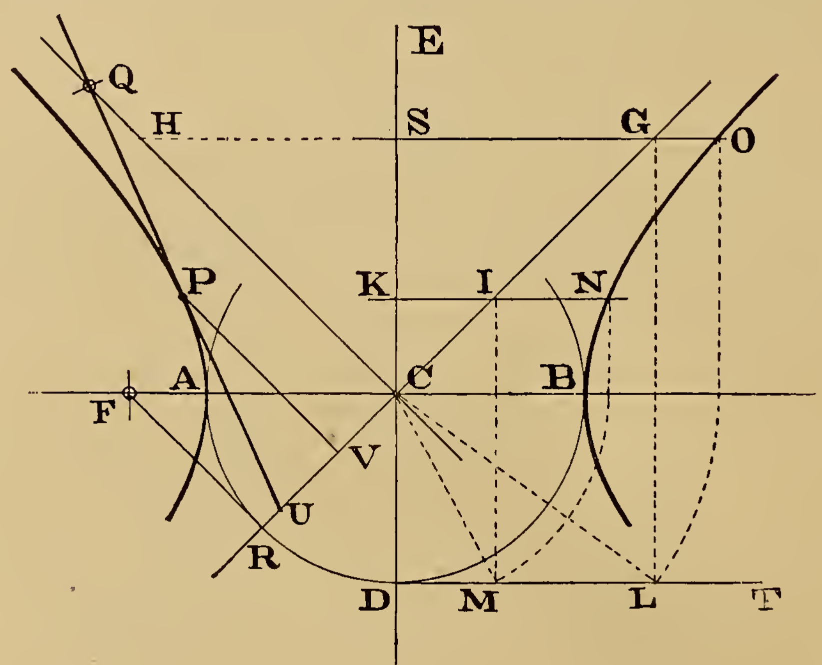

FIG. 48.

draw DCE perpendicular to AB, also DT tangent to the semicircle. Draw OS perpendicular to DE, and with radius OS describe about C an arc cutting DT in L; on SO set off $SG = DL$: then CG is an asymptote to the curve. Draw any line KI parallel to AB, cutting CG in I, and produce it; set off on the tangent, $DM = KI$, then on the prolongation of KI set off $KN = CM$, and N will be a point on the hyperbola.

To find the focus: Produce GC to cut the semi-circumference in R, at which point draw a perpendicular to GR; this will cut the line of the major axis in the focus F.

To draw a tangent at any point on the curve, as P: Draw PV parallel to the asymptote

HC, cutting the other asymptote *GR* in *V*. On *GR* set off $VU = CV$, and draw *PU*, which will be the required tangent.

To find the point of contact when the tangent is given: Suppose *PU* to have been drawn, mechanically (72), tangent to the hyperbola. Produce the given tangent to cut one asymptote, *GR*, in *U*, and the other one, *HC*, in *Q*. Bisect *QU* in *P*, which will be the point of tangency sought.

To draw the hyperbola through a given point, the asymptotes and the directions of the axes being known: This may be done by reversing the construction of Fig. 48. Thus, *CG* and *CH* being the given asymptotes, let it be required to draw the hyperbola through the given point *N*; the major axis to be horizontal and the minor axis vertical. Draw *NK* perpendicular to the minor axis *CE*, cutting the asymptote *CG* in *I*; through *I* draw a parallel to the minor axis, intersecting at *M* an arc described about *C* with radius equal to *KN*; through *M* draw a horizontal line, cutting the line of the minor axis in *D*: then CD = semi-major axis, and the curve can be completed as before.

78. Third Method.—Given, in Fig. 49, the major axis *AB*, and *P* a point in the required curve. Draw *PO*, *BE* perpendicular to *AB* and equal to each other, and join *PE*.

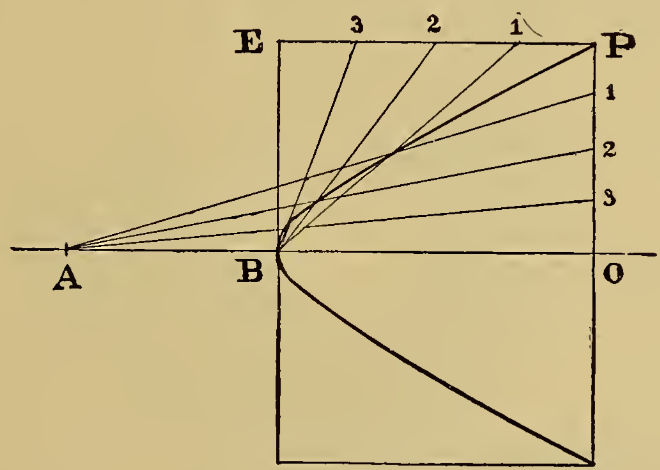

FIG. 49.

Divide *PO* and *PE* into the same number of equal (or proportional) parts, numbering the points of division from *P* on each line. From *A* draw lines to the points upon *PO*, and from *B* lines to the points upon *PE*: the intersections of the lines thus drawn to corresponding points of division, as for instance *A*1, *B*1, will lie upon the hyperbola required.

DRAWING OF ROLLED CURVES.

79. Rolled Curves, Roulettes, or **Epitrochoids,** are curves described by points carried by one line which rolls upon another, the latter being fixed. The line which carries the tracing-point is called the *generatrix* or *describing-line*, and the fixed one is called the *directrix* or *base-line;* either of these may be straight, or both may be curved.

The nature of perfect rolling contact may be best seen by a study of that which is not

perfect. The polygon in Fig. 50 rolls along the fixed right line with a hobbling motion. In the position shown, it is clear that if there is to be no sliding the point A must be at rest, and the motion indicated by the arrow must consist in a rotation of the whole figure about A as a fixed centre, until B reaches M, and the side AB coincides with LM. The polygon will then turn about B as a centre, and so on; its perimeter measuring itself off upon the right line. If the number of sides be increased, the hobbling will be diminished, and if the number becomes inconceivable, it will be imperceptible. The broken outline then becomes the dotted curve, to which the right line is tangent, and the change from one centre of rotation to another goes on continuously. But what is true up to the limit holds true at the limit, and the fact remains, that at any instant the point of contact is at rest, and every point in the figure is at that instant *turning about that point of contact as a fixed center.*

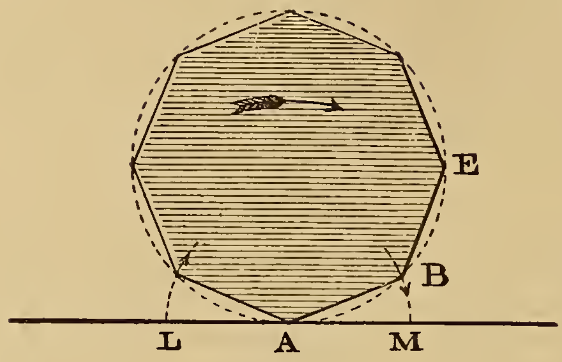

FIG. 50.

80. And upon this fact depends the readiest and most reliable known method of drawing the curves in question. In Fig. 51 AA is a curved ruler fixed to the drawing-board,

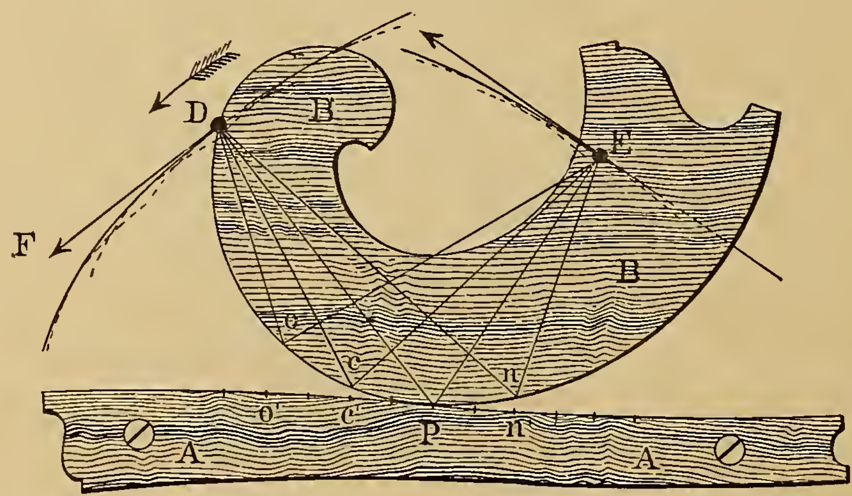

FIG. 51.

and BB is a free one rolling along it. Let a pencil be fixed to and carried by the latter, either in the contact edge as at D, or at any distance from it, as at E. Since P, the present point of contact, is the instantaneous centre of rotation, the point D is at the instant moving in the direction DF, perpendicular to the contact radius DP. Therefore DF is tangent to the path of D, traced as the ruler BB rolls; but it is also tangent to the circular arc whose centre is P and radius PD, consequently the path of D is also tangent to that arc.

Now, let the arcs Pc, Po, of BB, be equal to the arcs Pc', Po', of AA: then cD will be contact radius when c reaches c', and oD when o reaches o'. If then we describe, with these radii, circular arcs about c' and o', the curve traced by D will be tangent to those arcs; and that traced by E will be tangent to arcs about the same centres, with PE, cE, and oE as radii.

THE CYCLOID, EPICYCLOID, AND HYPOCYCLOID.

81. The Cycloid is traced by a point in the circumference of a circle which rolls upon its tangent.

In Fig. 52 find the length Aa' of a convenient fraction Aa of the circumference, (see **56**); step this off the required number of times upon the tangent, making AE equal to the semi-circumference. Divide each into the same number of equal parts, draw chords from P to the points of division on the semicircle, with which as radii strike arcs about the corresponding points on AE: the cyloid is tangent to all these arcs. The more numerous the arcs, the more perfectly is the curve mapped out.

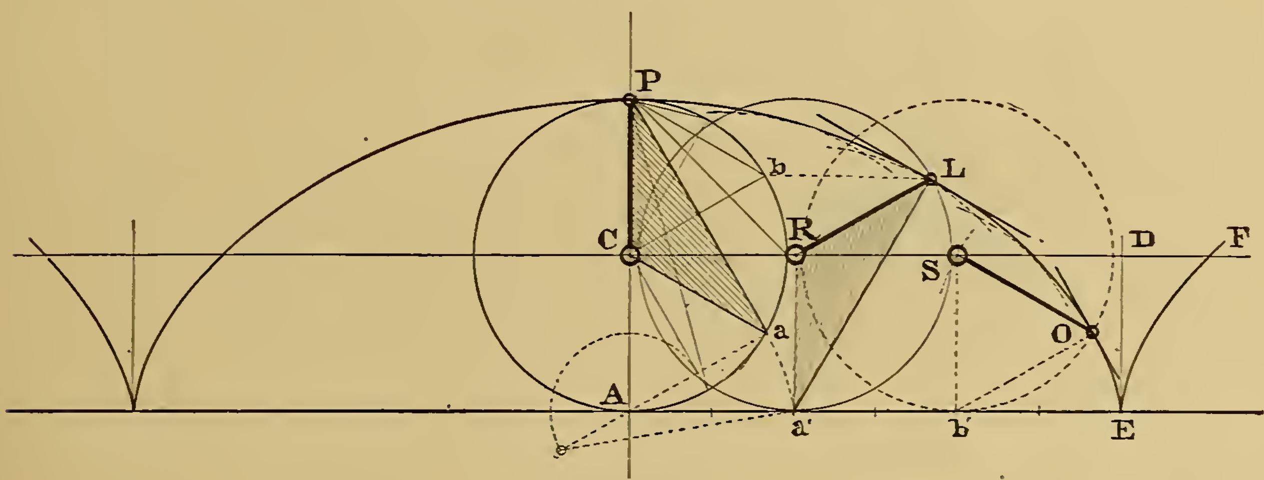

FIG. 52.

To find points on the curve: By the above process no points on the cycloid are located; the curve touches all the arcs, but the points of contact are not known; nor is it necessary, for the purpose of merely drawing the cycloid, that they should be. But if desired, they may be found thus: When aC becomes contact radius, it has the position $a'R$ perpendicular to AE; but the angle aCP remains unchanged. Therefore make the angle $a'RL$ equal to it; then RL is the new position of the *generating radius* CP, and L is a point on the cycloid. Also, $a'L$, the *instantaneous radius,* is normal, and a perpendicular to it at L is tangent, to the curve.

Conversely: Let O be any point on the curve; about this as a centre describe an arc with radius equal to CP. This arc cuts CD, the path of the centre of the rolling circle, in S: then OS is the generating radius; Sb', perpendicular to AE, is the contact radius; and $b'O$ is normal to the cycloid.

82. The Epicycloid.—The describing circle in Fig. 53 rolls upon the *outside* of another whose centre is G. Draw the common tangent at A, set off upon it the length of Aa (any convenient fraction of semi-circumference AP), and find (**56**) the arc of the base circle equal to that length. Step this off as many times as are necessary upon the base circle, making arc AE = semi-circumference AP. The curve is drawn by tangent arcs in the same manner as the cycloid.

The path of the centre C is in this case another circle whose centre is G, and the contact radii $a'R$, $b'S$ are prolongations of the radii Ga', Gb' of the base circle; which slightly

modifies the processes of finding the point of the generated curve corresponding to a given point of the rolling contact, and the converse.

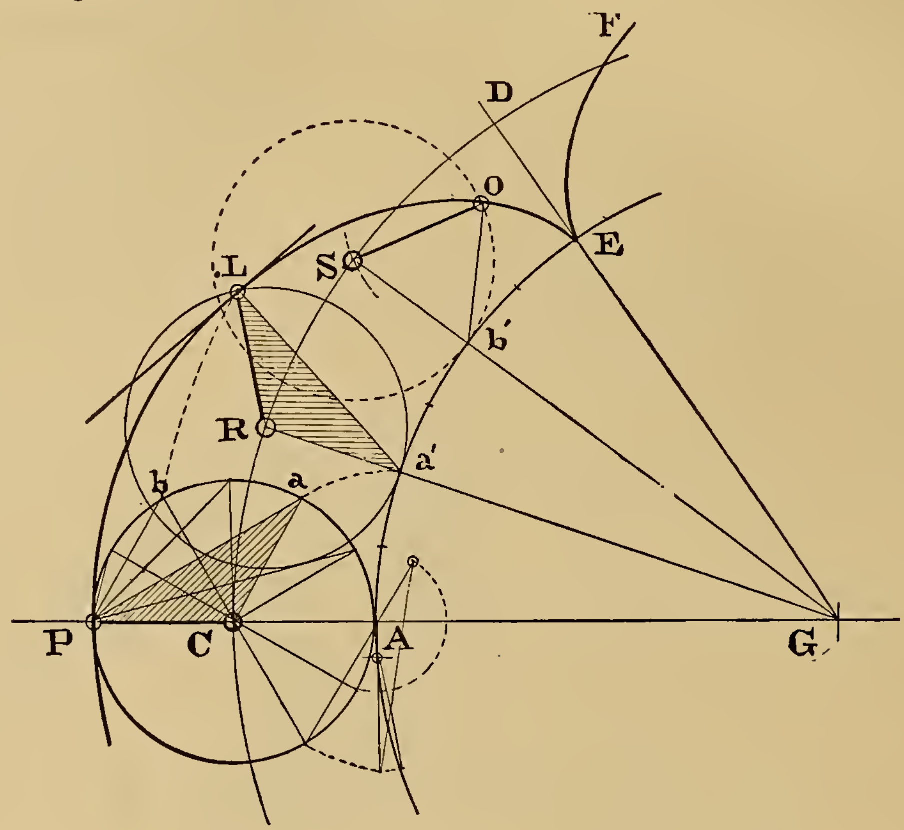

FIG. 53.

83. **The Hypocycloid.**—Traced, as shown in Fig. 54, by a point in the circumference of a circle rolling *inside* a larger one. The construction is in all respects the same as in

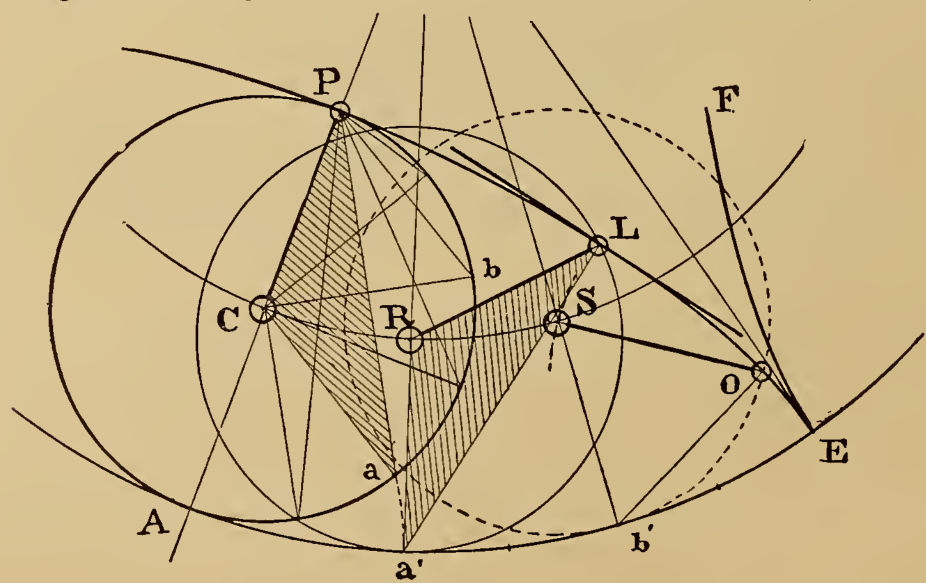

FIG. 54.

the case of the epicycloid, and as the diagrams are lettered to correspond throughout, no further explanation is needed.

In all three of these curves, if the rolling continues beyond *E*, a new branch springs up, which is perfectly symmetrical with *EL*.

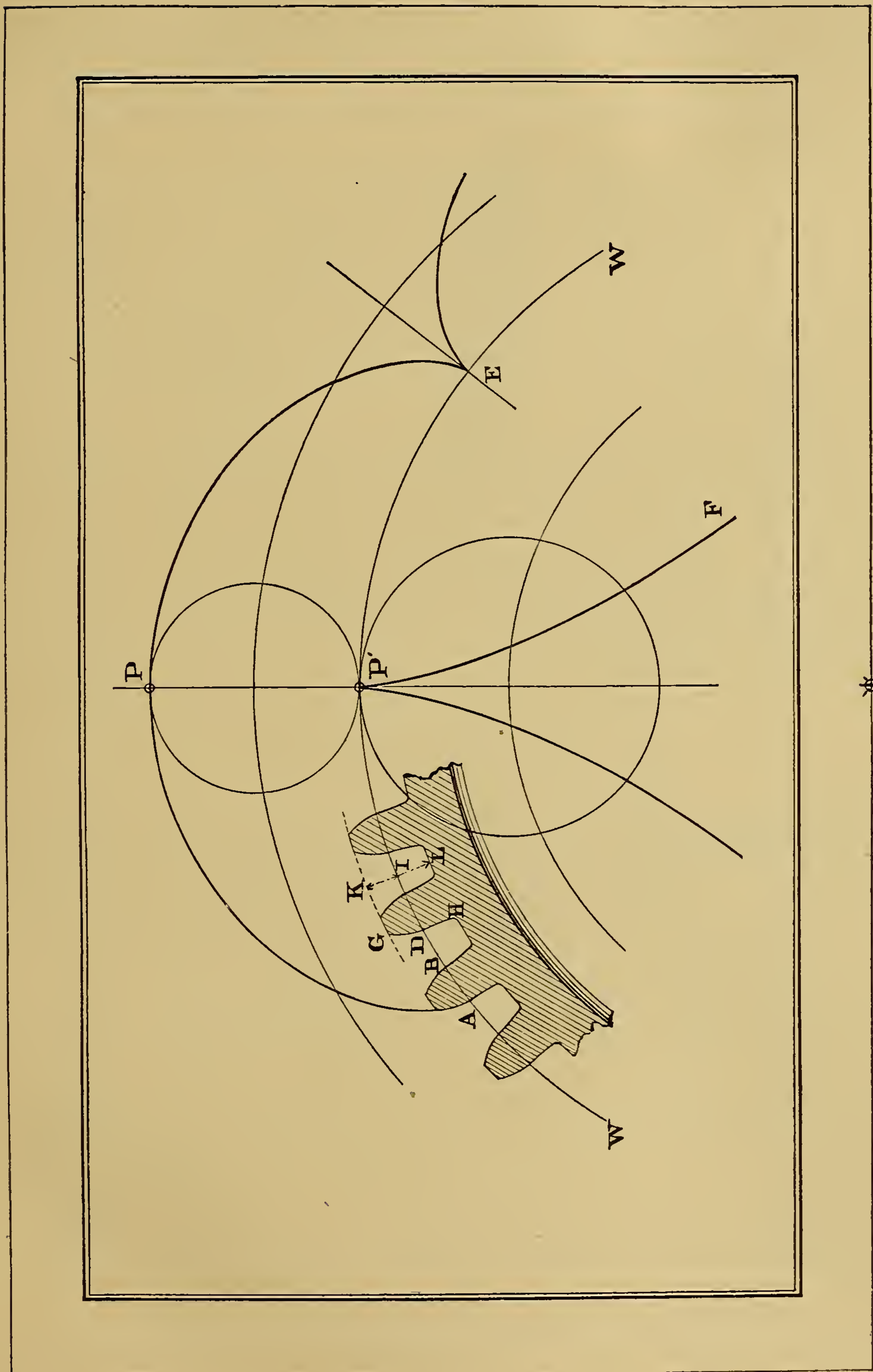

Fig. 55.

It is particularly to be noted that these branches are *tangent* to ED, and therefore to each other, at E; and that special care should be taken in constructing those parts of the curves which lie in this immediate neighborhood, not only because the change in the rate of curvature is here most rapid, but because they are the parts most frequently used in practical applications.

84. Spur-wheels with Epicycloidal Teeth.—Separate diagrams have been given in illustration of the epicycloid and hypocycloid, in order to make the construction clear. But these may be satisfactorily combined in one exercise, showing the application of these curves in forming the teeth of a wheel, as in Fig. 55. In laying out a spur-wheel, the first step is to draw the *pitch circle*, of which WW is a part; the circumference of this circle is then divided into as many equal parts as the wheel is to have teeth, which determines the *pitch arc* AD, embracing a tooth and a space; and the space BD should be a little greater than the thickness of the tooth AB: the difference, say $\frac{1}{24}$ to $\frac{1}{20}$ of the pitch, is technically called *back-lash.*

The *face*, DG, or part of the tooth which lies outside the pitch circle, is, as shown, a part of the epicycloid APE; the flank, or part which lies inside, is a portion of the hypocycloid $P'F$; and the describing circles for these may be of equal diameters or not, at pleasure. But if two wheels are to gear together, the same describing circle must be used for the faces of the teeth upon one wheel and the flanks of those on the other, and *vice versa.* The whole height of the tooth, LK, may be from $\frac{3}{4}$ to $\frac{4}{5}$ of the pitch, LI being greater than KI in the proportion of say six to five.

85. It is seen then that in laying out the teeth of wheels only small portions of the epicycloids are used; so that it is not necessary that the divisions of the describing circles used in constructing those curves should be aliquot parts of their circumferences. But it is necessary to have equal arcs set off on all the circles employed, and this may be very readily done as shown in Fig. 56. On the tangent at A, set off by the scale, $AB = 1$, $BC = 3$, taking care that AC does not exceed the radius of the smallest circle.

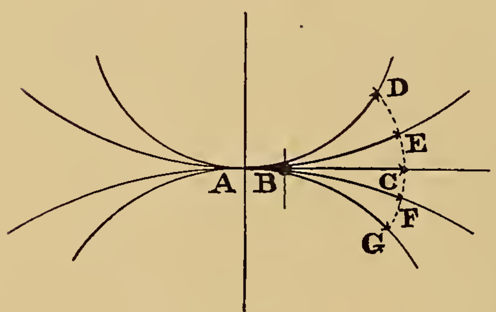

FIG. 56.

Then with centre B and radius BC describe an arc cutting the four circles, thus determining the four arcs AE, AD, AF, AG; which being each equal to AC (**56**), will be equal to each other.

86. The curves forming the outline of one side of a tooth having been accurately constructed, and carried well beyond the limits of the proposed height, the teeth may be very expeditiously drawn by the device shown in Fig. 57. A mould or template of the exact form of the contour turns freely about a fine sewing-needle fixed in the drawing-board at the centre of the wheel. The edges of the teeth being set out on the pitch circle, the template is set to coincide with each of these points in succession, until the left-hand sides of all the teeth are drawn, when the template is reversed in order to draw the right-hand sides.

Such a template may be made of thin wood like white holly, to which the contour is easily transferred as explained in (**71**). But one equally serviceable may be made with less

labor of Bristol-board, upon which the epicycloids may be constructed directly; the pitch circle, and the centre line of the wheel as well as its centre, should be marked in fine lines, after which the working edge of the template may be cut nearly to the line with a sharp

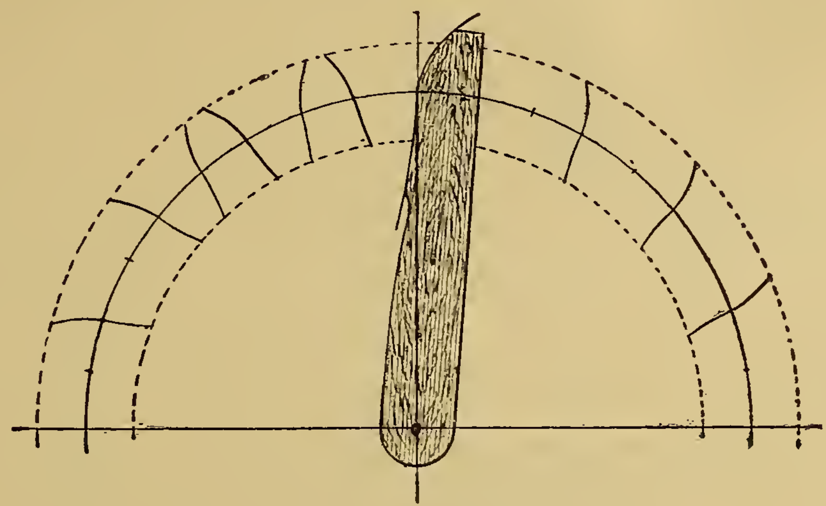

FIG. 57.

knife. Then shellac the template on both sides, and when dry it can be worked to the precise contour with a file and sand-paper. To use such a template with the pen, gum a bit of card to the back, a little away from the edge, so as to keep it slightly raised off the surface of the paper.

THE INVOLUTE OF THE CIRCLE.

87. The Involute is in a sense the converse of the cycloid, being generated by the rolling of a tangent line upon a circle. Thus in Fig. 58 the ruler, carrying in the line of its edge the pencil P, while rolling around the cylinder describes the curve under consideration.

It may also be regarded as generated by unwinding a fine inextensible thread from a cylinder: the thread being always taut and always tangent to the circumference, its length is always equal to that of the arc from which it was unwound. Thus, beginning at O, make the tangents BE, AP, DF, respectively equal to the arcs OB, OBA, $OBAD$, and so on: then the curve passes through the extremities E, P, F, G, etc., of these tangents; intermediate points may be found in like manner. The tangent OG, then, will be equal to the circumference of the circle, and if the unwinding be continued, a spiral will be formed; the distance between the successive coils, measured on the tangent to the base circle, as for instance EH, being constant and equal to the circumference.

Considering the curve as traced by the rolling of the ruler: the point of contact, for instance A, is the instantaneous centre (**79**, **80**); therefore AP is normal to the curve at P, and, in general, the normal at any point of the involute is tangent to the base circle, and *vice versa.*

If the ruler continue to roll in the direction of the arrow, it is clear that after P reaches O, a new branch will be formed as shown by the dotted line; the two branches being tangent

to each other, and to the radius *CO* at its extremity. The method of (**80**) may be applied also to the involute, which is obviously tangent to arcs described about *A*, *B*, *D*, etc , as centres, with radii *AP*, *BE*, *DF*, and so on ; by subdividing the base circle more closely,

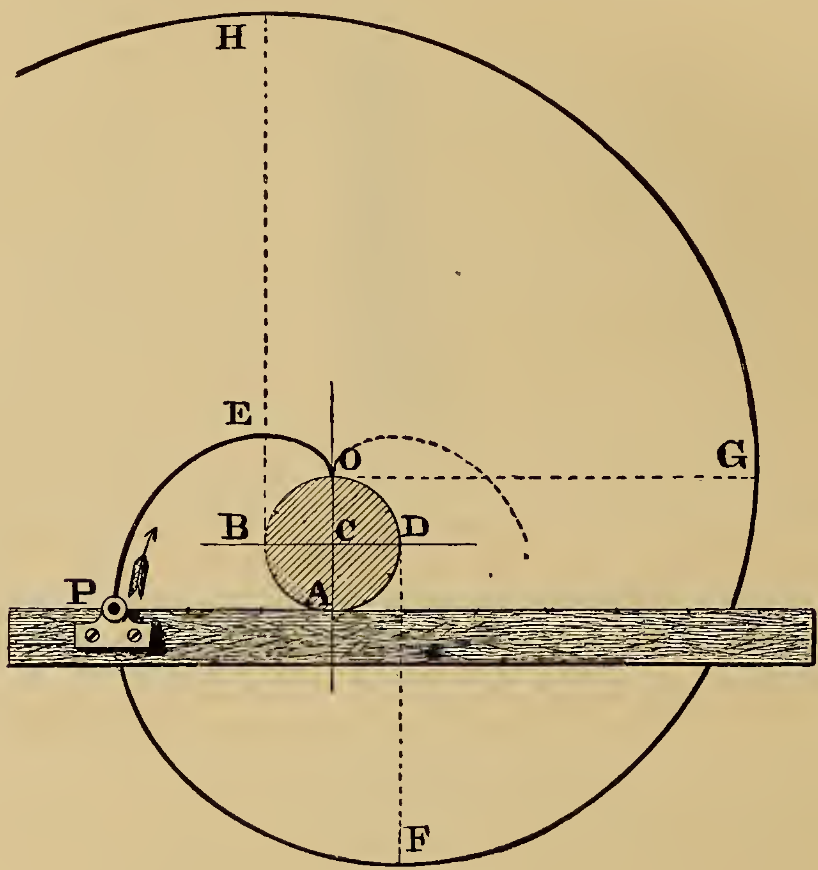

FIG. 58.

the curve may in this manner be very accurately mapped out, without locating any points upon it.

THE EPITROCHOID.

88. Although the term epitrochoidal is used in a general sense, including all rolled curves, yet custom sanctions also a special sense ; and the curve traced by the rolling of one circle upon another, when the marking point is not situated upon the circumference, is the one ordinarily meant when "*the epitrochoid*" is simply mentioned.

In Fig. 59, if the circle whose centre is *C*, roll upon the circle whose centre is *G*, carrying a marking point *P* situated without the circumference, it describes the looped curve *PLE*, called the *curtate epitrochoid.* If the tracing point be situated within the circumference as at *V*, the resulting waved curve *VWX* is called the *prolate* epitrochoid. Both may be drawn by the method of tangent arcs. Since, as in the preceding cases, the rolling circle measures itself off upon the base circle, the position of the generating radius can always be found as in the construction of the epicycloid, and, its length being constant, points

on the curve are readily determined. Also, the instantaneous radius being always normal to the epitrochoid, the tangent at any point can be drawn with the same facility. For example, let it be required to draw the tangent at L; with a radius equal to CP, describe about L an arc cutting the path of the centre C in the point R, and draw RG, cutting the base circle in a': then $a'R$ is the contact radius, RL is the generating radius, the instantaneous radius $a'L$ is the normal, and a perpendicular to it is the required tangent.

If the circle which carries the marking point rolls on the inside of the base circle, the resulting curve is called, by way of distinction, an *internal* epitrochoid; if it rolls upon its tangent, the curve is called simply the *trochoid;* but as the only modifications in the above process are those due to the form of the base line, no further explanation is necessary.

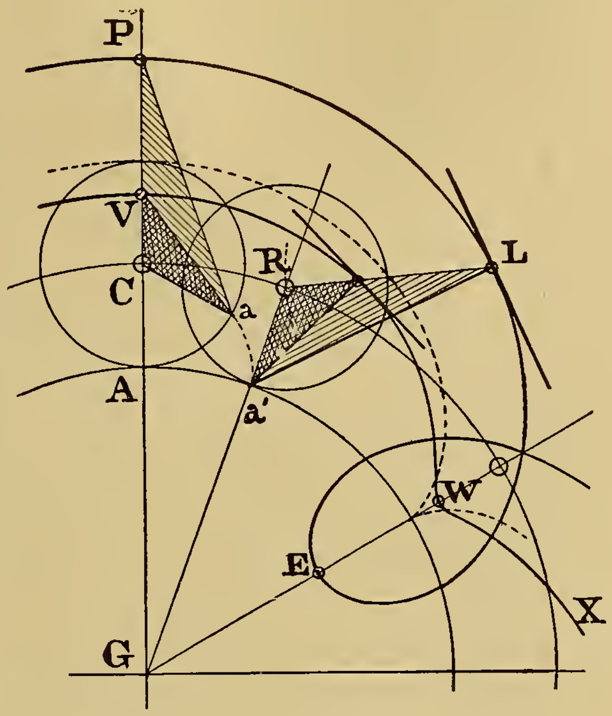

FIG. 59.

SPIRALS.

89. A Spiral is a plane curve traced by a marking point moving along a right line, while at the same time the right line revolves about one of its points as a fixed centre.

This fixed centre is called the *pole*, and a right line drawn from it to any point of the curve is called a *radiant*, or *radius vector.* Supposing the angular velocity of the revolution to be uniform, the linear velocity of the marking point along the radius vector may be governed by any law at pleasure, and thus an infinite variety of spirals may be produced.

Some of these may also be otherwise generated, as already shown in the case of the involute of the circle, in which the tracing point moves uniformly along the tangent, while the tangent revolves uniformly about the centre; but the definition first given is perfectly general, and includes them all.

90. The Archimedean Spiral.—The radius vector revolving uniformly, let the tracing point travel along it with uniform velocity. The resulting curve is the well-known spiral of Archimedes, also called the *equable spiral,* because the rate of expansion is constant, so that the distance between any two consecutive coils, measured on a radiant, is the same.

If this distance or rate of expansion be given: With it as radius, describe a circle about the pole P, Fig. 60, and divide its circumference into any number of equal parts by radial lines; divide the given distance PA into the same number of equal parts, and set out from the pole P, upon consecutive radii, as I, II, III, IV, distances equal to one, two, three, etc., of these subdivisions: the spiral is then drawn through the points thus determined.

If any two radiants and their included angle be given: Divide the included angle and the difference between the radiants into the same number of equal parts; then set off on the

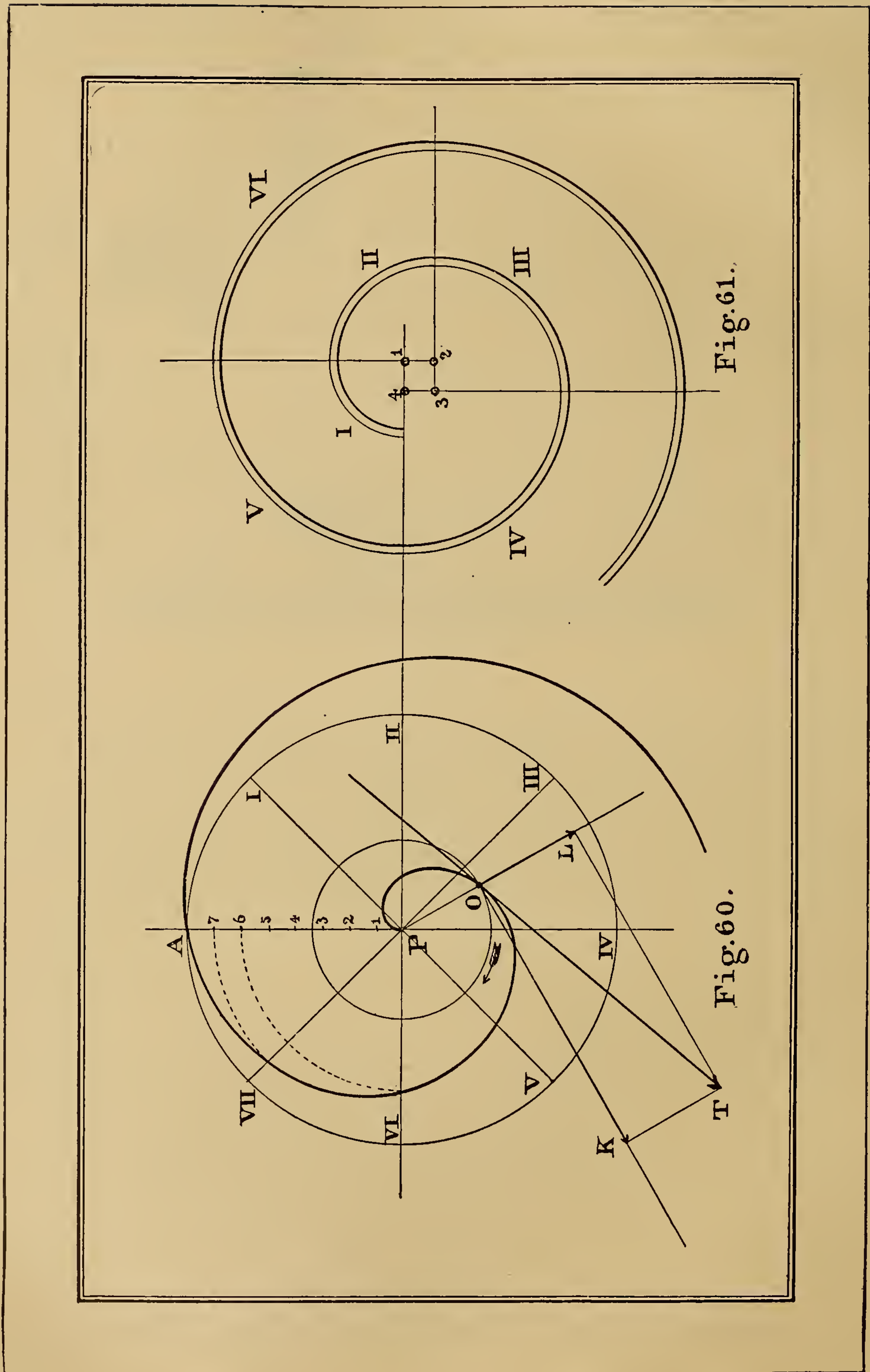

Fig. 60.

Fig. 61.

first dividing radial line a distance equal to the least radiant plus one of the subdivisions of the difference, and so on; each radiant being greater than the preceding by an amount equal to one of these subdivisions.

To draw a tangent at any point, as *O:* Drawing a circle about *P* through *O*, and rectifying a convenient fraction thereof as in (**56**), the length of the circumference may be found; and the radial travel per revolution, or rate of expansion, is known. Then, the arrow indicating the direction of the revolution, set off on the radiant *OP* a distance *OL*, and on a perpendicular to it a distance *OK*, making *OK* : *OL* :: circumference through *O* : rate of expansion. Complete the parallelogram, and the diagonal *OT* is the required tangent.

91. Pseudo-spiral.—The above spiral is sometimes used in forming the outlines of cams, in mechanical movements, and for such purposes it must be accurately laid out. It is also the form assumed by a coiled spring such as the mainspring of a watch or clock. But for the purpose of representing such a spring, it is quite needless to make so laborious a construction; an approximation to the true curve is easily drawn as shown in Fig. 61. The sides of the small square being produced, limit the quadrants I, II, III, IV, of which the centres are 1, 2, 3, 4; then the same centres are again used in order, 1 for quadrant V, 2 for VI, and so on: thus the distance between the successive coils is equal to the perimeter of the original square. A still closer approximation may be made by using a hexagon, or an octagon, in like manner. It is obvious that this more nearly represents the involute of the circle than the true equable spiral; but for the purpose mentioned the curve would not in fact extend very near to the pole, in the mechanism itself; and the accuracy of this method is amply sufficient for making the drawings of such springs.

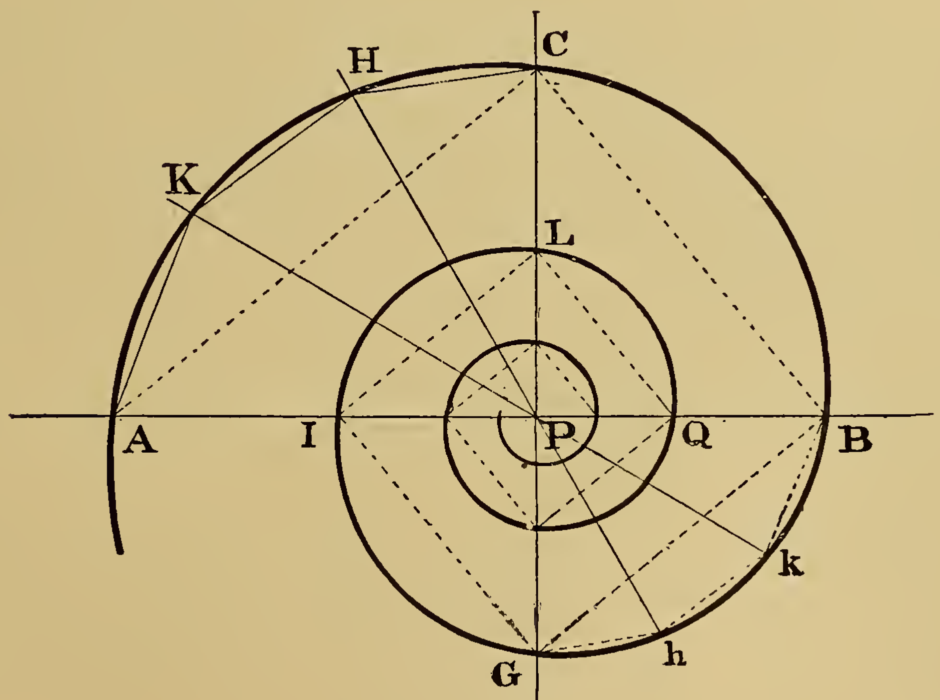

FIG. 62.

92. The Logarithmic Spiral.—In this curve the successive radiants which include equal angles form a series in *geometrical* progression, each being greater or less than the preceding in

a certain constant *ratio*: in the Archimedean spiral such radiants form a series in *arithmetical* progression, each being greater or less than the preceding by a constant *difference*. From the above definition it follows that the radiant which bisects the angle between two others is a mean proportional between them. Thus in Fig. 62, if the angles APK, KPH, HPC are equal, we shall have

$$AP : PK :: PK : PH,$$

$$PK : PH :: PH : PC, \quad \text{and so on.}$$

Had the radiants AP, PH and their included angle, then, been given, the intermediate point K would have been found by bisecting the angle APH, and setting off PK, a mean proportional between the two given radiants. Had a *diameter*, as AB, and the pole P, been given, the process would have been the same—this angle of 180° is bisected by PC perpendicular to AB, and PC is a mean proportional between the segments AP, PB of the diameter.

93. *To find the mean proportional between two given lines*: Set off from any point P on any right line a distance PA in one direction equal to one of the lines, and in the opposite direction a distance PB equal to the other (Fig. 63). Upon AB as a diameter describe a semicircle, intersecting at C a perpendicular to AB at P: then PC is the mean proportional required.

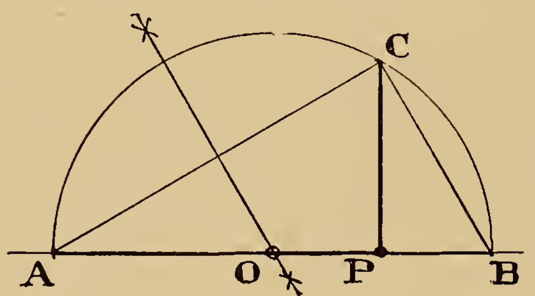

FIG. 63.

To find a third proportional to two given lines: In Fig. 63 let AP be one of the given lines, and PC, perpendicular to it, the other. Draw AC and bisect it by a perpendicular cutting AP in O, about which point as a centre describe a semicircle through A and C: this will cut AP produced in a point B, and PB is the third proportional required; for by this construction we have $AP : PC :: PC : PB$.

This last construction is often useful in extending the spiral; thus, had the curve ACB in Fig. 62 been determined, it is clear that PG, on the prolongation of CP, must be such a third proportional to CP and PB.

94. The length of PG may be otherwise determined; for since when G lies on the spiral we must have $AP : PC :: PB : PG$, the triangles APC, BPG will be similar, whence BG must be parallel to AC. So also GI is parallel to CB, IL to BG, LQ to GI, and so on.

This principle is capable of more extended application; for producing the radiants KP, HP to cut the spiral in k, h, it will be apparent that the chords KH, kh are parallel; so also Bk is parallel to AK, hG to HC, and so on.

95. Another Method of setting off divisions in geometrical progression is shown in Fig. 64. Draw the two lines OM, ON, including any convenient angle, and also AB, cutting them both. About O describe the arc AC, and draw CD parallel to AB; then about O describe the arc DE, and draw EF also parallel to AB; similarly, make $OK = OB$, draw KL parallel to AB, and so on as far as may be desired in each direction.

If now the distances from O to the points of division on either line be set out from the pole, on successive radiants including equal angles, the curve thus determined will be a logarithmic spiral.

This construction may be also adapted to any special case in which two radiants and their included angle are given. Thus, had *OA*, *OB* been made equal to those radiants, the lengths of others would be found as above; but in constructing the curve, care must be taken that the angles between them are made equal to the given angle.

MECHANICAL METHODS OF PLOTTING CURVES.

96. Mention has already been made of the use of a slip of paper for locating points upon the ellipse (**62**). Analogous expedients are often available in other cases, enabling the draughtsman to work at once more neatly and more rapidly. Thus, if the divisions determined as in Fig. 64 be set off on the radial edge of the strip of paper shown in Fig. 65, it is not necessary even to draw the radiants of the logarithmic spiral; a circle is described about the pole, and a fine sewing-needle, passed through the corresponding point *O* of the paper strip, is fixed in the drawing-board at the centre of the circle, whose circumference is first subdivided. Then swinging the strip round, setting its edge by each point of division in order, the proper point of the spiral is marked with a finely sharpened pencil. By making the divisions of the radial edge equal, the Archimedean spiral can be laid out with the same facility.

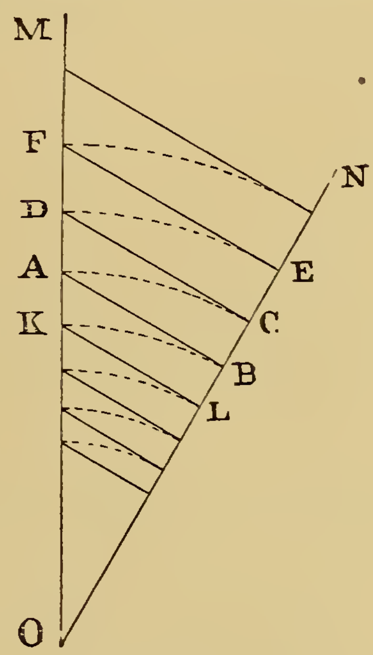

FIG. 64.

97. In Fig. 66, the lower edge of the paper strip is tangent to the circumference of the circle, and the subdivisions upon it are equal in length to the corresponding subdivisions of that circumference. Turning this as before about a needle fixed at the centre *O*, setting the radial edge *OR* by the successive points on the circumference, and marking with a fine pencil-point the corresponding divisions of the lower edge (counting from zero to the *left* when the rotation is in the direction of the arrow), the curve

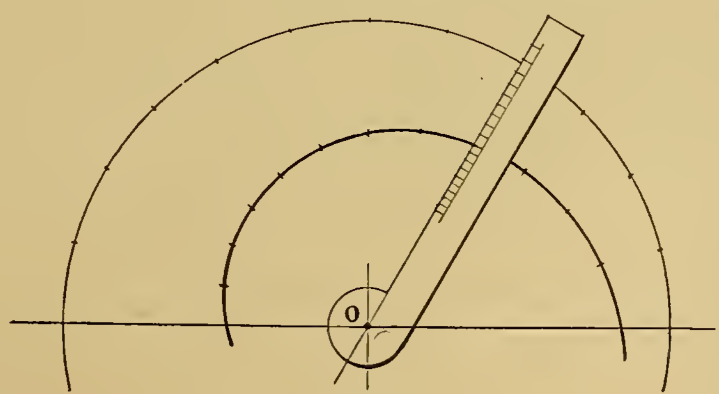

FIG. 65.

thus determined is the involute of the circle *ABC*. By counting from zero to the *right*, the direction of the rotation being unchanged, points are determined in the curve *ADE*, the *companion to the involute*. Of these two curves, the former is traced by unwinding an inexten-

sible fine thread from a *fixed* cylinder; the latter by unwinding it from a cylinder which turns upon its axis so that the thread is always tangent to the circumference at the same point.

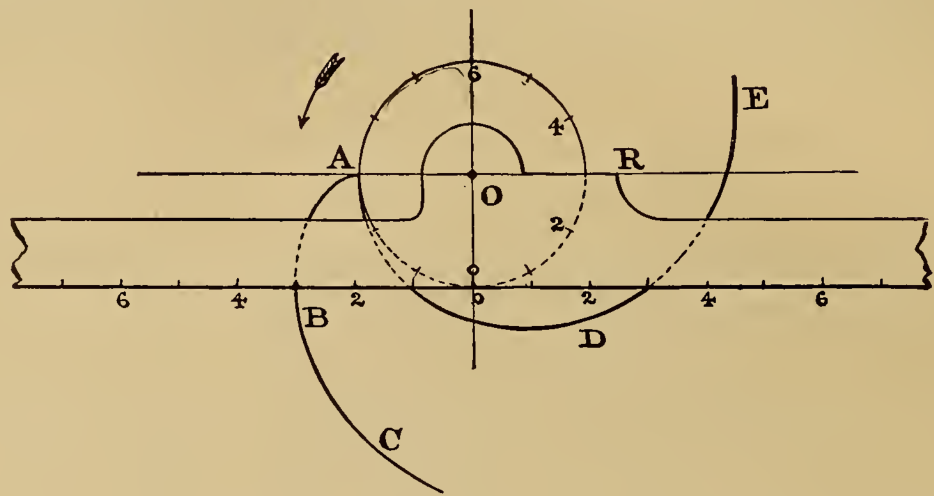

Fig. 66.

But the above simple device shows that they may also be defined as the paths of points travelling uniformly along a right line, while the line itself revolves uniformly about a fixed point not situated upon that line.

These, however, are special cases, the travel along the line during each revolution being equal to the circumference of the circle to which the line is tangent. But it need not be; and by means of different graduations of the paper strip it is easy to plot, without drawing a single construction line except the divided circumference, this whole class of spirals, of which the involute and its companion are the most conspicuous representatives.

98. The path of any point on the connecting-rod of a direct-acting engine may be plotted as easily as the ellipse, the only difference being that one of the fixed points on the paper slip is to be kept always on the circumference of the circular path of the crank-pin. If the connecting-rod is coupled to two equal cranks, like the side rod of a locomotive, and these cranks turn in opposite directions, the middle point of the rod traces a lemniscate; which may also be easily plotted by the above method.

The piston-rod of an oscillating engine passes always through a fixed point, which may be represented by a fine sewing-needle stuck into the drawing-board: against this let the edge of a straight slip of paper be pressed, a fixed point thereon being kept on the circumference of the path of the crank-pin, and the curve described by any point on the rod can be plotted; if this circle be of infinite radius, the curve becomes the conchoid, while if the centre of oscillation be located on the circumference, it becomes the limaçon.

99. Any of these paths may be constructed at pleasure as exercises in the drawing of curves; and as a final illustration of this method of manipulation, we give in Fig. 67 three of the curves which may be plotted by means of Newton's Square.

This was devised by that renowned geometer for drawing the cissoid; of which the generating circle is that whose centre is O in the figure. On the vertical line through O a needle is fixed at E, the distance OE being equal to the diameter of the circle. Against the needle the side BF of the square DBF is kept, while the point A, whose distance from B is equal to OE, is kept always on the horizontal line through O. Then the middle point P of

AB traces the cissoid PC; a point D, so situated that $AD = AP$, traces the waved curve called the wizard, while the corner B traces the looped curve called the sprite.

It will be clear that when P reaches C, the working edge DB will coincide with the

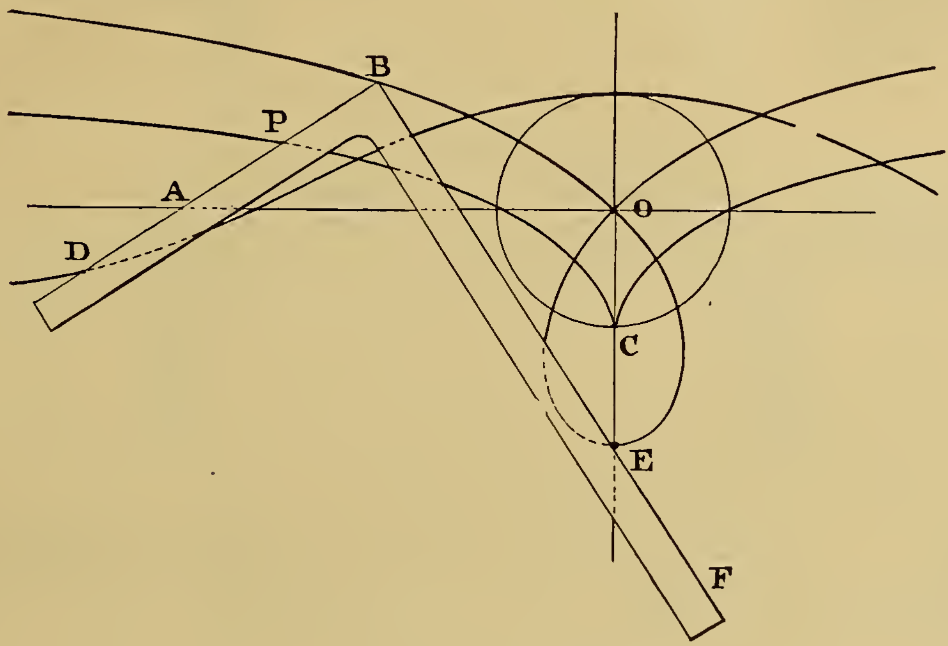

FIG. 67.

vertical line OE: the square must then be turned over, and the under side used for continuing the curves to the right.

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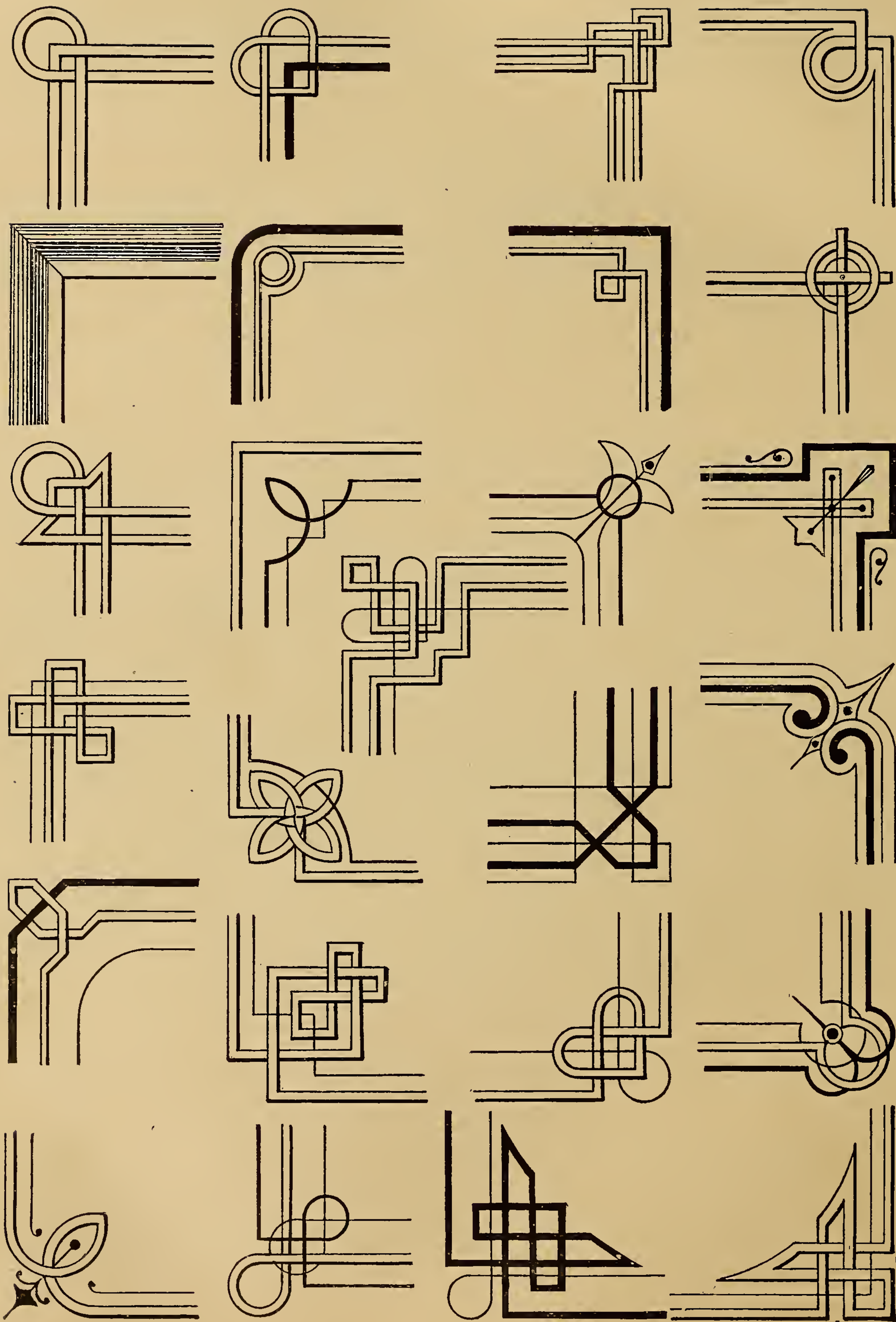
100. Before proceeding farther, we give on page 52 a number of examples of "corner-pieces," suitable for borders of a more ornate style than the simple ones heretofore employed.

From these the ambitious student may select such as suit his fancy or his estimate of his skill,—with the understanding that they are to be executed with the same care and subjected to the same criticism as any part of the exercises contained within them. With this understanding, no instructor possessed of ordinary common-sense will attempt to control his selection, except by hints and suggestions. He may and very likely will at first set his heart upon something glaringly inappropriate to the case in hand; but he will be much more readily convinced of this after he has drawn it than before. And good taste in this can no more be acquired without experience, than can perfection in the execution of the exercises themselves.



CHAPTER III.

THE PRINCIPLES OF PROJECTION.

101. Pictorial representations, such as photographs and perspective drawings, while they may give perfectly correct ideas of the form, proportions, and general appearance of the objects depicted, are of no use for constructive purposes. Parallel lines are represented by converging ones, equal lines by unequal ones, right angles by those which are obtuse or acute: so that, in general, no accurate information can be obtained by direct measurement.

Working drawings, however, are needed for the absolute guidance of the workman, who by their aid alone must be enabled to produce precisely what is intended, and that only. Such drawings, then, must exhibit the exact forms and dimensions, and also the true relations of the various parts, in a manner both direct and unmistakable.

The difference between a perspective and a working drawing is sometimes expressed by saying that the former shows things as they *seem,* the latter as they *are:* but the precise nature of this difference may be explained as follows:

102. Suppose the eye (regarded as a single point) to be placed in a given position in front of a plate of glass, and to be looking perpendicularly against its surface, which is called the **picture plane.** Any point of an object beyond the glass is visible, because light is reflected (or **projected**) from that point to the eye in a right line, called a **visual ray.** And the point in which that ray pierces the picture plane is the representation, or **projection,** of the point from which it came. If a sufficient number of such points be found, the outlines of any object may be fully determined, and the drawing thus made will present to the eye, if placed in the position originally assigned to it, the same appearance as the actual contour of the object itself.

103. When the eye is placed at a *finite* distance from the picture plane, the drawing is said to be in **perspective.** In this case the visual rays converge, and but one of them can coincide with the axis of vision, which is perpendicular to the plane. Nevertheless, the eye, without straining and without turning, can receive distinct impressions from all visual rays embraced in a cone whose angle at the vertex is about sixty degrees: the picture plane cuts from this cone a circle, within which the drawing should be included.

104. Other things being equal, the visual rays will converge the less rapidly the farther the eye is removed from the picture plane. If the eye be placed at an *infinite* distance, the rays will become parallel to each other and to the axis of vision, which is still supposed to be perpendicular to the plane. In these circumstances the drawing is called an **orthographic projection,** the visual rays are usually designated as **projecting lines,** and the picture plane is called the **plane of projection.** It will readily be seen that a plane may be so interposed as to cut the parallel projecting lines at any other angle; that is to say, the eye, although infinitely remote, may not be directed perpendicularly toward the plane: the drawing is then

called an **oblique projection,** and in some cases is of great utility and convenience. The former arrangement, however, is much more extensively used, and thorough familiarity with it is necessary to an intelligent use of the latter. For the present, then, it will be understood that in speaking of **projections** reference is made only to those constructed on the orthographic system.

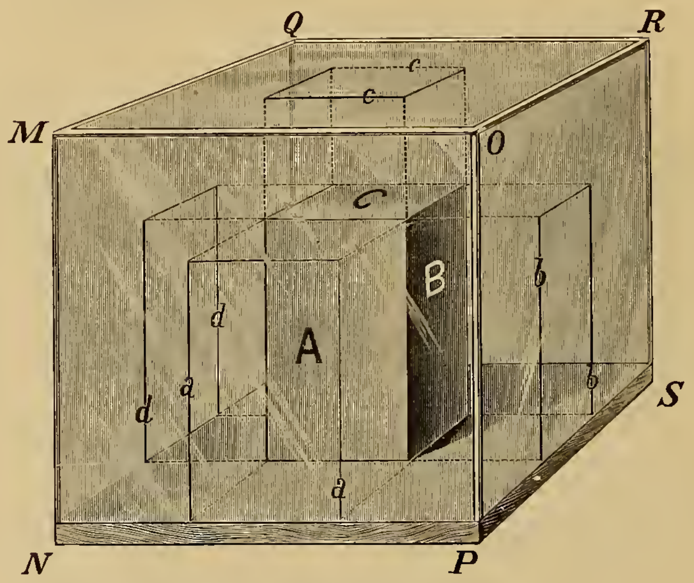


FIG. 68.

105. Fig. 68 is a perspective drawing of a rectangular prism, placed within a case formed of plates of glass, upon which the projections of the object are made in the manner above explained: thus, *aa* is the projection of the front face *A*, *bb* is that of the side face *B*, and *cc* is that of the upper face *C*. The left side face of the prism is not visible, but its projection upon the left side of the glass case is shown at *dd*.

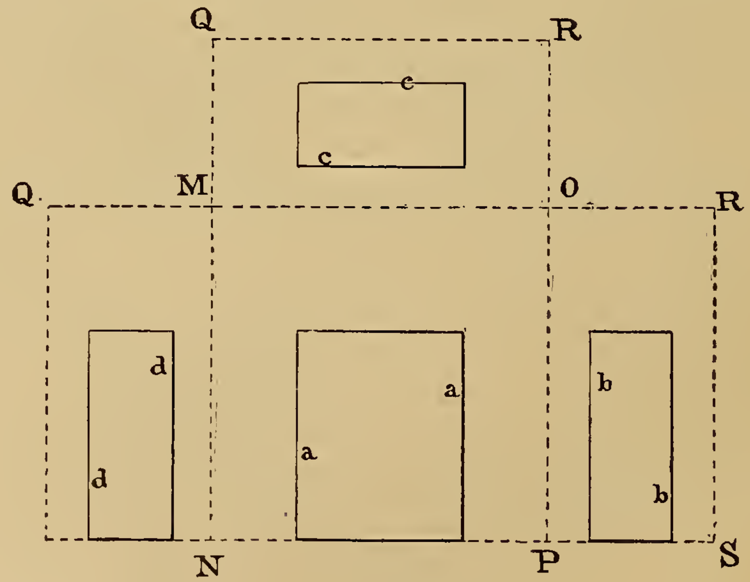


FIG. 69.

Now suppose the sides and the top of the case to turn outward, about the lines *MN*, *OP*, and *MO*, until they coincide with the front plate *NO*. The four projections will then be arranged as shown in Fig. 69, the dotted lines representing the sides of the glass case. Re-

garding A as the front of the object, the projection a is commonly called a **front view**, b and d are called **side views**, and c is called a **top view**; although in the case of an object lying upon its longest side a may also be properly called a **side view**, b and d being then spoken of as **end views**. These three projections, which are made upon vertical planes, are also, particularly in relation to architectural subjects, designated as **elevations**; c being in that case called a **plan view**, or simply a **plan**.

106. In reference to the arrangement, it is to be specially noted that b, which is a view of the object *from the right*, is placed **at the right** of the front view a, while d, a view *from the left*, is placed **at the left**. This may seem of small consequence, and in fact it is so in representing so simple an object; but suppose A to be for example a side view of a locomotive heading to the right, and it will readily be seen that the view of the headlight and the cow-catcher, which is only to be had by looking at the engine from the right, is most naturally and appropriately placed at the right side of the front view: as the parts seen in that end view are thus brought nearer to their representations in the first one. It may be said that without any reference to planes of projection, this arrangement of the views is that which would be suggested by simple common-sense to any one who, provided with paper and pencil, should sketch an object as he sees it, first from directly in front, and then from the right or the left, as the case may be.

107. In regard to the plan, or top view, however, there is this to be said, that no ambiguity or doubt can arise if it be placed below the elevation, or front view. Because it is invariably a view of the object *as seen from above*. Under no circumstances should a projection be made which involves the idea of the eye being placed beneath the object and looking upward,—with the possible exception of a design for a frescoed ceiling. Such projections *are* occasionally made, and are even recommended by some: but that does not lessen the force of the objection, that they are difficult to read because they represent things as seen from a most unusual direction. It is certainly often necessary to know the appearance of the lower end of a part of a machine, for example, which is to be placed in a vertical position. But in order to find it out, one does not hold the object over his head and look upward: the natural proceeding is either to turn it upside down, or to lay it on its side and look at it horizontally; and the same course may be adopted in making the projections,—of which more hereafter.

Consequently, a top view may be placed **either above or below** the elevation, as may appear most convenient or suitable; in the case of the locomotive, for instance, it is easy to see that the top view, being of nearly uniform breadth, will present a more satisfactory appearance if placed under the side view, in which usually the rail is represented, than if put so far above as to be well clear of the smoke-stack, sand-box, cab, and steam-drum, which form a very irregular contour.

108. In Fig. 70, A, B, and C are the projections of the three faces of the prism correspondingly lettered in Fig. 68. It is to be noted that each projection exhibits only two dimensions; thus the front view gives the height and the breadth, the side view gives the height and the thickness, the top view gives the breadth and the thickness. Also, that the height ad is the same in A and B, the breadth ab is the same in A and C, while the thickness ac is the same in B and C.

It will also be observed that while the perspective representation conveys in one view a correct and complete idea of the form of the object, no one, nor yet either two, of the projections are sufficient to do this. Were *A* and *B* only given, it could not be told whether any or all of the vertical edges were square or rounded off; and if *A* and *C* were the only views, the top or the bottom, or both, might be either plane, or curved transversely; but the three together define the form completely.

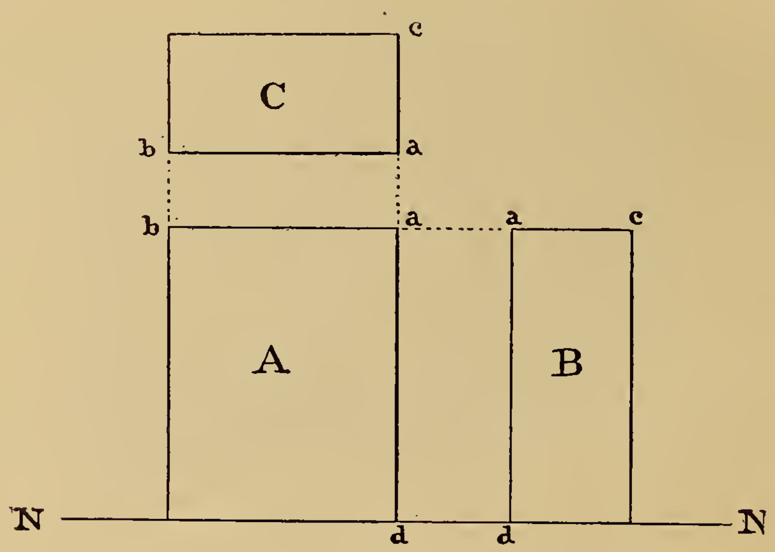


FIG. 70.

109. Again, it will be noted that each of the projections represents a face of the prism in its true form and size. A very little study of Fig. 68 will show that this is because the planes of projection are parallel to those faces; the projection of a line is made by connecting the projections of all its points, from which it follows that if a line be parallel to the plane of projection, it will be represented on that plane by a similar and equal line. For example, had the object been an upright cylinder, the projections *a*, *b*, and *d* would still have been rectangles, but *c* would have been a circle. The position of the object is of course arbitrary; thus, the prism might have been turned part way round, or placed upon an edge or a corner. But the position actually shown is the simplest one, and the one which would naturally be selected for making a **working drawing**, which Fig. 70 is properly called, since by its aid the object represented can be made by a workman, who takes his measurements directly from the projections.

The prism is shown as resting upon the horizontal plane *NS*, forming the bottom of the glass case. This in Fig. 70 is indicated by the horizontal line *NN*, called a **base line**; which, though not essential, is in many cases introduced with excellent effect, for it at once shows unmistakably that the view beneath which it appears is an elevation, and not a plan.

110. This position of the prism, which may be also described as that in which its lines and faces are parallel and perpendicular to the paper in each of the projections shown in Figs. 69 and 70, has been selected not only by reason of its simplicity, but because, as will be shown presently, it is the starting point from which we set out in determining the projections of the same object in oblique positions. In those positions the projections, though still orthographic, may not show the lines and angles in their true dimensions; and in order to

construct them correctly, it is necessary to possess the information conveyed by Fig. 70, the construction of which is indeed an essential preliminary.

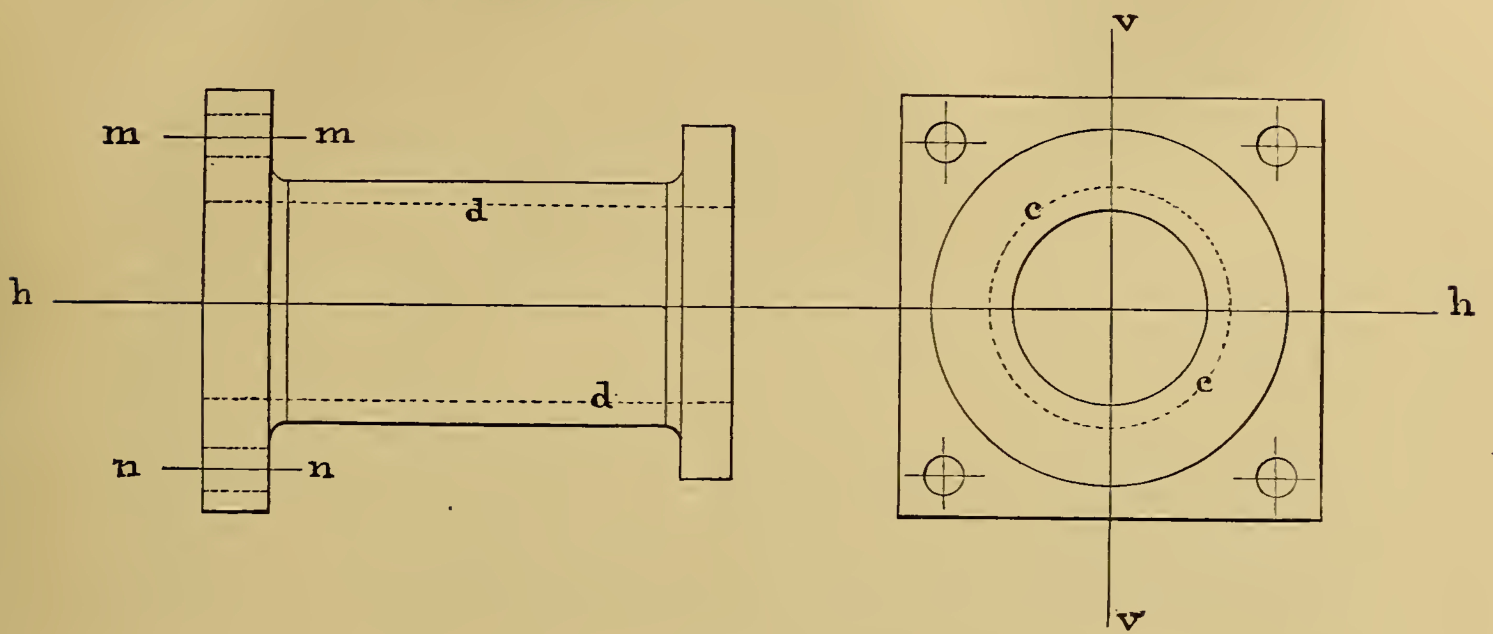


FIG. 71.

Before going on to discuss those more complicated matters, however, it may be remarked that in a working drawing the most simple and direct representation is always the best; so that the principles already explained are sufficient for practical application, to very good and useful purpose, in the delineation of minor mechanical details: of which accordingly we introduce one or two examples.

111. Fig. 71 represents a horizontal cylinder, having at one end a circular flange, and at the other a larger square one, in which are four bolt-holes.

In this case two views are sufficient, as a top view would be identical with the side view. The circular flange on the right-hand end being the smaller, a view from that end defines the whole, as the square one at the left is visible beyond, as also are the bolt-holes. Parts not visible are indicated by dotted lines, as for example *d* in the front view and *c* in the end view, and the outlines of the bolt-holes in the former.

In Fig. 70 it will be noticed that the projecting lines *aa*, *bb* are represented, in dotted lines, for the purpose of indicating the relations between the different projections. In illustrative diagrams, this is necessary in order to aid the reader in following the explanations. But when the subject is mastered they are no longer required, and in working plans are very rarely introduced, the practised eye being able to *read* the drawings, and compare one view with the others, without such assistance.

112. But when, as in Fig. 71, the object is symmetrical about an axis, and especially if it be a surface of revolution, it is customary to connect such views as these by a continuous **centre-line**, as *hh* in the figure; and in the end view to locate the centre by a transverse centre-line, *vv*. Similarly, the centres of the bolt-holes are marked in the end view by short vertical and horizontal centre-lines, corresponding horizontal ones, *mm*, *nn*, being drawn in the side view. These are of the first importance, since it is by measurements from these lines

that the mechanic lays out his work, and they should never be omitted. Being imaginary lines, they should be drawn as fine as possible, and should **never terminate in a bounding line** of the object represented, or of any part of it. In drawings of machinery, if different colors are used, it is customary to draw them in red.

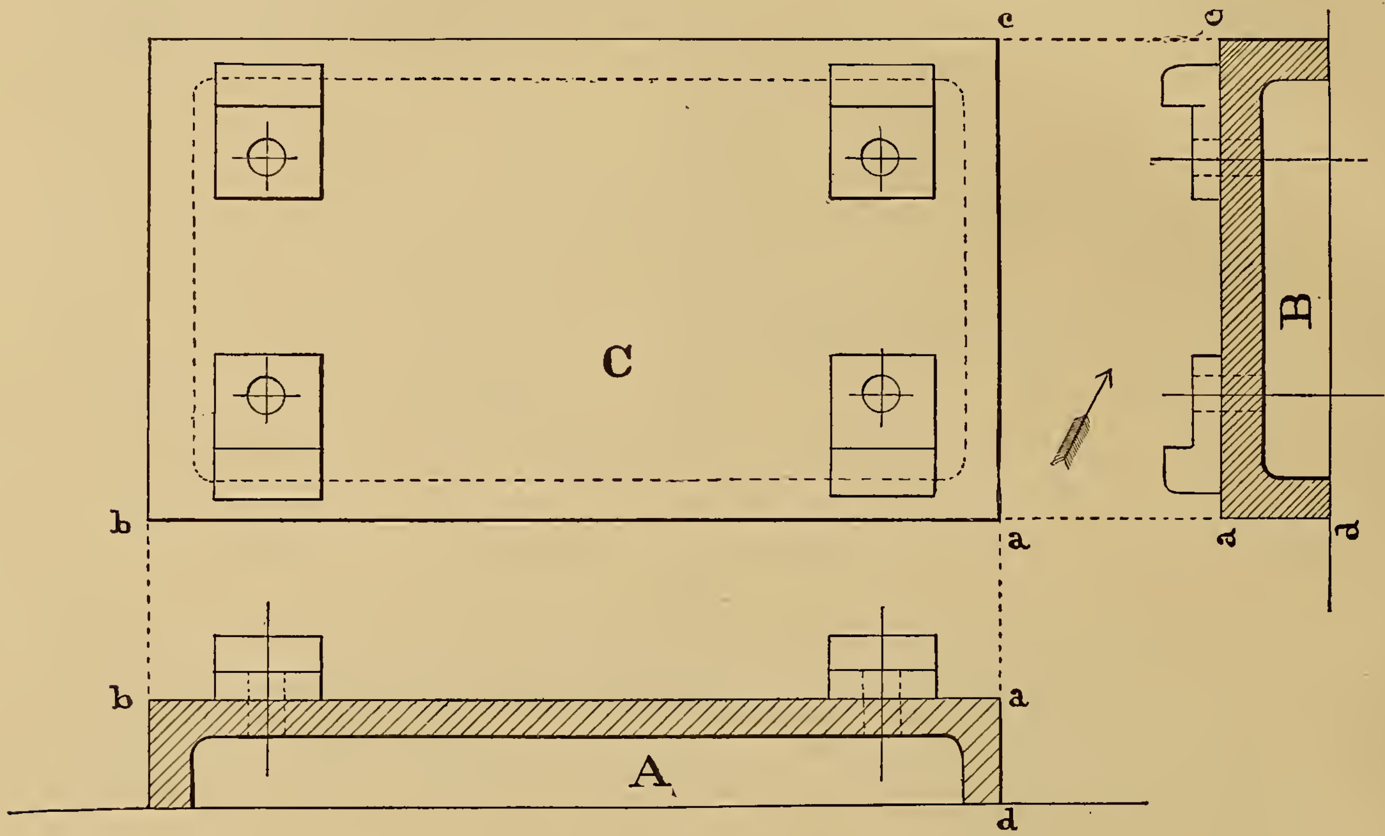


FIG. 72.

113. Fig. 72 represents a small bed-plate, with four facing-strips and lugs on its upper face. Though not absolutely necessary in so simple a subject, the views *A* and *B* are shown in section, as showing more distinctly the box-like form of the bed-plate: such views are commonly called **sectional elevations.**

In this instance three views are necessary; and the figure shows an arrangement of them which is frequently adopted in order to economize space. The breadth being considerable as compared to the height, it is clear that if, as in Fig. 70, *B* had been placed at the right of *A*, the three views would have occupied more space than they now do. But when it is placed at the right of the plan, as in the figure, it will be at once seen that *C* is as correctly a top view of *B* as it is of *A*.

When this arrangement is adopted, the fact that *B is* an elevation should be emphasized, by the proper use of shadow-lines according to the direction of the light shown by the arrow, as well as by the manner in which the dimensions, if any, are figured; so that it shall always be looked at as it should be, that is, by turning the paper a quarter round to the right, thus changing horizontal to vertical, and *vice versa.* If as in this example the subject is such as to render the introduction of a base-line appropriate, it forms a most effective aid to the correct reading of the drawing.

114. Figs. 70 and 72 are similarly lettered, and by comparing them a clearer idea may

perhaps be gained of the correspondence of dimensions in each two of the three projections. It is now time to impress upon the reader that advantage should be taken of this correspondence in constructing the drawings. Thus, for instance, the lines *ba* in the top view *C*, and *ad* in the end view *B*, should be drawn at one setting of the T-square; also *ac* in the top view and *ad* in the front view *A*, should be drawn at one setting of the triangle, and so on.

This is in accordance with the principle previously enunciated, that as much as may be should always be done with one adjustment of an instrument. But a great additional advantage of thus **setting up the different views simultaneously** as far as possible, lies in the fact that errors, to which even the most experienced are liable, are thus more likely to be at once detected than if each view were constructed separately. This is of special importance in planning, aside from the more perfect conception of the proposed structure gained by aid of the different views; but the saving in time is also considerable, and the habit should be formed at the start and always retained.

115. And so should the habit of **working from centre-lines**, which in general should be among the first lines pencilled in. For example, the lines *hh* and *vv* should be first laid down in drawing Fig. 71; equal distances above and below *hh* are laid off to locate the centre-lines *mm* and *nn* of the bolt-holes in the left-hand view, and the half diameter of each bolt-hole is set off by the scale on each side of its own centre-line; nothing could be less workmanlike, or more likely to lead to errors sooner or later, than to construct such drawings by measuring from the edge of a flange to the side of a bolt-hole, and from the side of one hole to the side of another. In short, the draughtsman should lay out his work on paper very much as the mechanic lays out his on metal.

116. In Fig. 73 are given three views of a piece formed by milling the upper part of a vertical cylinder into the form of a hexagonal prism. The top view is first made by drawing a circle of a diameter equal to that of the cylinder, and inscribing within it a hexagon. The points *a*, *b*, *c*, etc., represent the vertical edges of the prism, which in accordance with what precedes, will in the front view appear perpendicularly under these points, and be seen in their true lengths, as *al*, *ck*, etc. The edges *al*, *dg* coincide with the visible outlines of the cylinder, *lp*, *gh*, which also are represented in the top view by the points *a*, *d*. The upper front edge *bc* being horizontal, and parallel to the paper in the front view, appears of its true length in both the views *A* and *C*. But *ab*, *cd*, though horizontal and consequently seen in their true length in *C*, are inclined to the paper in the front view, where they appear what is technically called **foreshortened**, that is, of less than their actual length.

A third projection of so simple a thing is not necessary; but if as an exercise it be required to make it, we have now all the data: the altitudes will be the same in *B* as in *A*, the diameter x of the cylindrical part is also of course the same, and the "short diameter" y of the hexagonal part is given in *C*, and will be equal to that in the new view; to which it is transferred by setting off $\frac{1}{2}y$ on each side of the vertical centre-line.

117. Foreshortening, then, means the apparent reduction in the length of a line caused by viewing it obliquely. It was already understood that a line parallel to the paper is projected in its true length, while one perpendicular to it appears as a point. In intermediate positions it must therefore appear of intermediate lengths, and from Fig. 73 it will be clear that the amount of foreshortening depends upon the angle which the line makes with the paper, being

the greater as that angle approaches 90°. For example, the line *cd* appears shorter in the view *A*, where the angle is 60°, than in the view *B*, where it is 30°.

A little study also shows that in the former view the lines *ab* and *cd*, and in the latter the lines *cd* and *de*, are equally foreshortened, because they make equal angles with the paper.

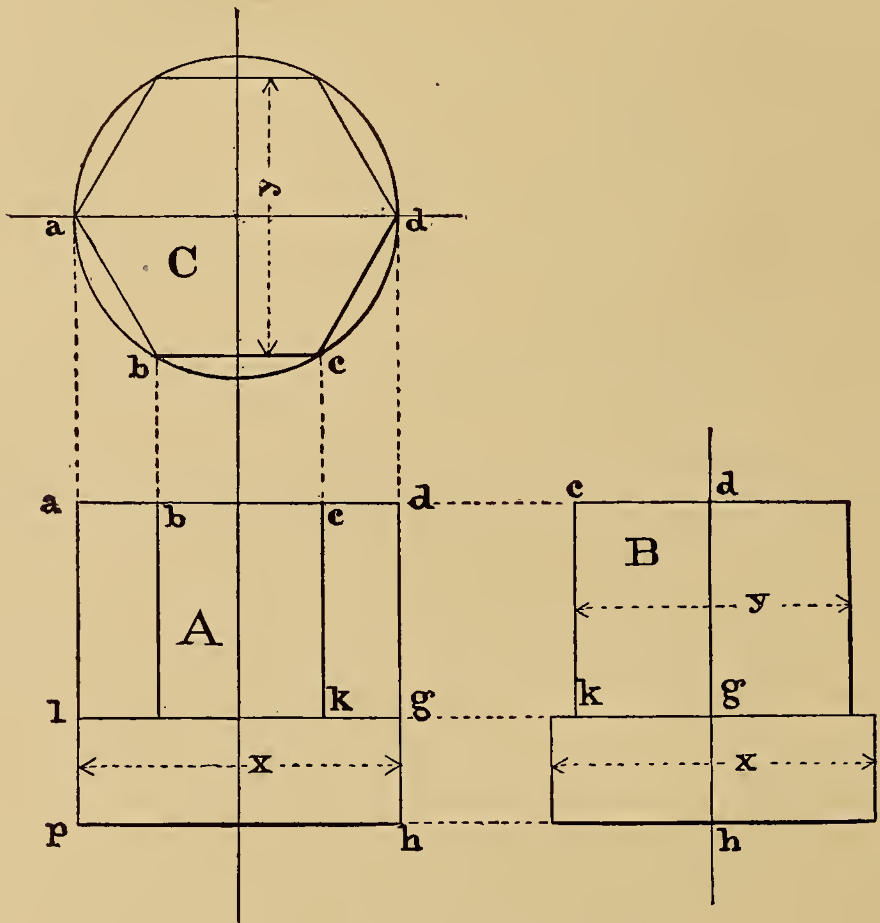


Fig. 73.

From this follows the deduction, which in subsequent operations will be found of great importance and utility, that **lines which are equal and parallel in space will appear equal and parallel in any projection**;—a fact which the student should fix in his mind so that it will stay there.

118. It will have been inferred that if two projections are sufficient to represent the object completely,—that is, so that by their aid a workman can make it and cannot make anything else,—no more are necessary. Ordinarily, two are not enough, while three are—but not by any means in all cases; and when they are not, as many more must be added as the occasion demands.

Fig. 74 represents a rectangular prism with projecting pieces upon its different faces, purposely so formed and located that they cannot, even by dotted lines, be so defined by means of the three views thus far considered, as to convey clear and unmistakable ideas of what is intended. In addition to the views *A*, *B*, and *C*, a view *D* of the left face is placed at the left side of the front view *A*. But a back view is also necessary; and it may, in conformity with the foregoing methods, be logically placed as shown at *E*. For regarding *B* for the

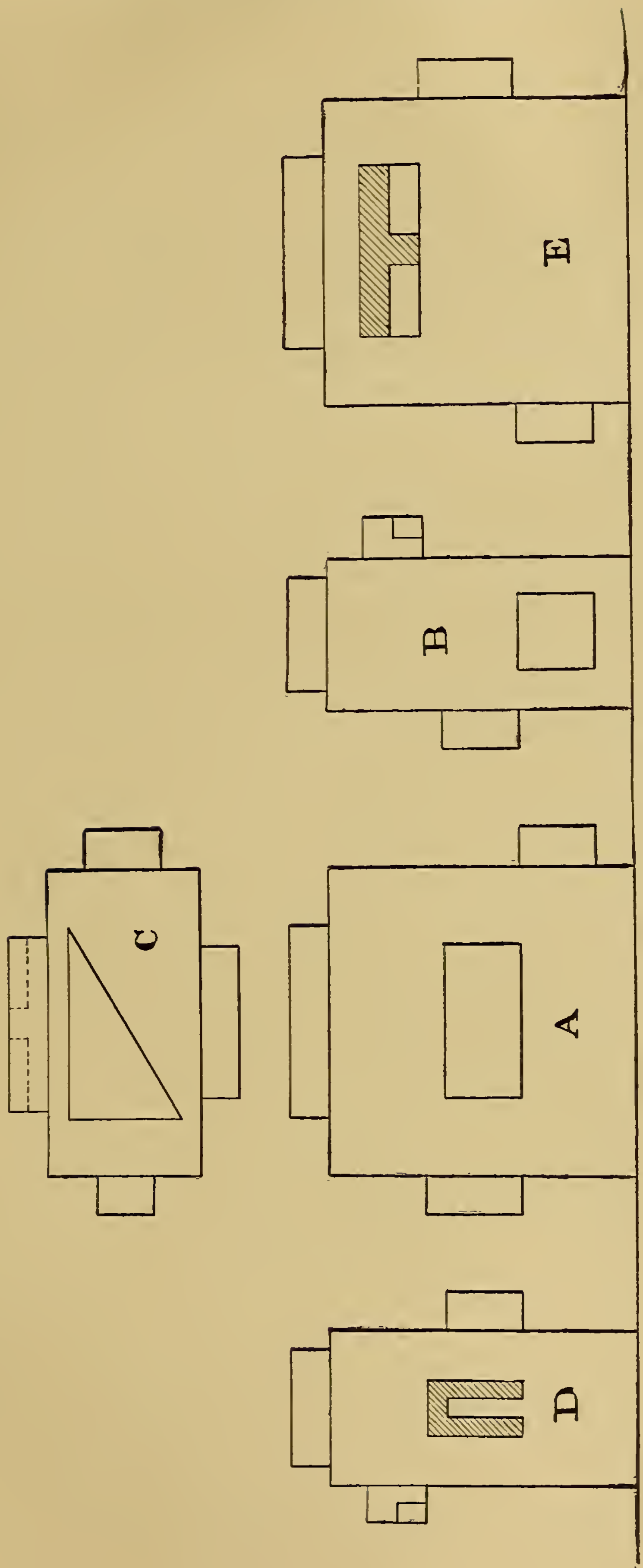


FIG. 74.

moment as a new front view, it will be clear that A will be a view of the object from the left, and E a view of it from the right.

119. Again, it sometimes happens that although a certain number of views are sufficient to define the construction, and that too without ambiguity when once they have been studied out, yet there is difficulty in reading them. If this difficulty can be lessened, and the meaning more clearly expressed, by the addition of other views, it is the draughtsman's duty to add them. We are entering upon the study of projections for a definite practical purpose, that is, the making of working drawings for workmen to use. They cannot use the drawings if they cannot read them; and if they cannot read them easily, then in a practical sense the drawings are bad in proportion to their obscurity, no matter how accurate or how perfect in execution.

CHAPTER IV.

PROJECTIONS CONTINUED.—OBJECTS IN INCLINED POSITIONS.

120. In Fig. 75 a rectangular prism is shown, as before, in its simplest position. Using a geographical illustration, A is a view from the south, D a view from the west: let it now be required to make a view from the south-west, as indicated by the arrow m. It will be readily seen that the horizontal plane on which the prism stands will be represented by a new base-line $N'N'$, perpendicular to the arrow; and that the vertical edges of the prism will be represented by lines ad, be, etc., drawn through the points a, b, etc., of the view C, parallel to the arrow: since the altitude is not affected by thus changing the point of view, these edges still appear in this new front view, A', of their true length, just as they did in the original front view A.

121. This is as though the glass case in Fig. 68 had been turned part way round *to the right*, while the prism within it retained its original position. The same result may be reached by supposing the case to remain stationary, while the prism, still standing upright on the bottom of the case, is turned *to the left* through the same angle. This is shown in Fig. 76: it needs no argument to show that the upper face of the prism will undergo no change of form or size, as seen from above, by this rotation about a vertical axis, so that the top view in this figure is simply a copy of C in Fig. 75. Also, the base-line NN remains the same, as well as the altitude of the prism, so that the front view A' is constructed by drawing the vertical edges ad, be perpendicularly over the points a, b, which represent them in the new top view, and similarly for the other edges; the heights being the same as in views A, D, of Fig. 75; and the views marked A' in Figs. 75 and 76, clearly, are identical.

122. Now let it be required to construct a view of the prism in its new position, Fig. 76, as seen from the right; or, geographically, from the east, as indicated by the arrow k. It may be, perhaps, more clearly seen how this is done, by at first adopting the arrangement of views employed in Fig. 72. The horizontal plane will be represented by a new base-line $N''N''$, perpendicular to the arrow; turning the paper round so as to make this line horizontal, it will be obvious that the vertical edges will be represented by ad, be, etc., drawn through the points a, b, etc., of the top view, parallel to the arrow k, and of the same length as in the front view A': which is in fact their true length, because they are parallel to the paper in this view also.

If this new view B'' be turned a quarter round, and placed at the right of A', it will appear as B'; which is the arrangement most frequently adopted.

123. The student should now be able without difficulty to perceive that the view B' may be at once constructed, without the use of B'', as follows: Draw in the top view a line pp, parallel to NN: this represents the edge of a vertical plane, in this instance containing the front vertical edge of the prism; as though the object had been placed in contact with the front plate of the glass case in Fig. 68. To an observer placed at the east and looking toward

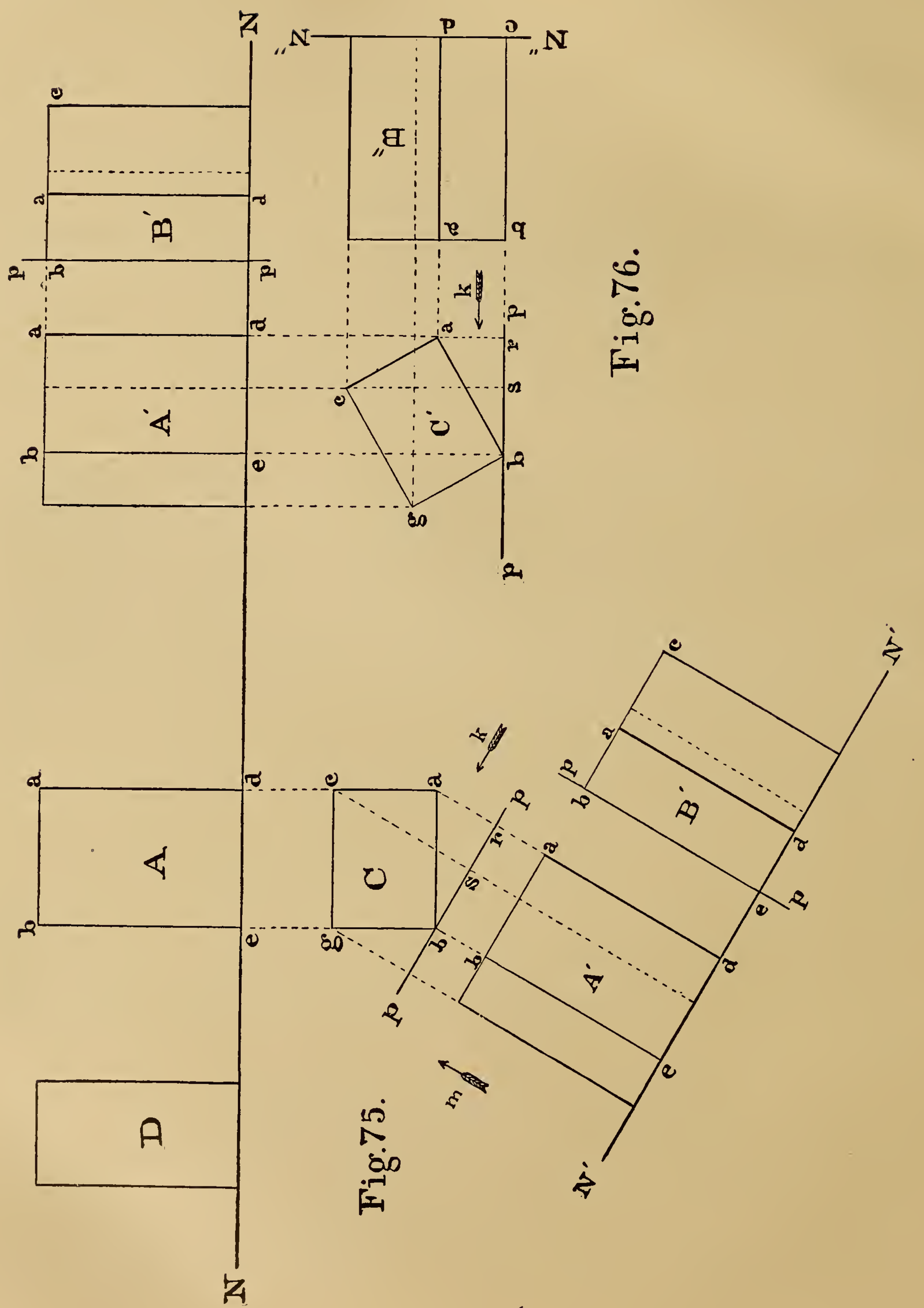


Fig.75.

Fig.76.

the west, as shown by the arrow k, this plane will also present its edge, and appear as the vertical line pp in the view B'. To such an observer, also, the edge represented by b in the top view will appear at the extreme left, and coincident with pp; while the edge represented by c will appear at the extreme right, and at a distance from pp equal to cs; to which therefore bc in the required view B' is made equal. Similarly, the perpendicular distance of the edge ad from the plane pp is ar in the top view; and as ar is seen in its true length, whether we look down upon it vertically from above, or look at it horizontally from the right of the object, this length is set off as ba in the new view.

124. This last mode of procedure, evidently, would be decidedly the preferable one were it desired to supplement the front view A' in Fig. 75, by a side view as seen from the direction indicated by the arrow k. This has been done in the illustration, and it is now perfectly obvious that B' is the same in Figs. 75 and 76. In short, the only difference between these figures consists in the fact that in the former the original view C is made to do double duty, being also the top view in the group C, A', B', in reading which the paper should be so held as to make the new base-line $N'N'$ horizontal.

Virtually, then, in Fig. 75, and actually in Fig. 76, the original position of the object has been changed by rotation about a vertical axis, that is, one perpendicular to the paper in the top view; and it is to be noted, that not only is *that top view unchanged*, but so also are the altitudes, or *measurements parallel to the axis:* and it is by reason and by means of these two circumstances that we are enabled to construct the projections of the object in its new position.

125. Now, referring to Fig. 68, we may suppose the prism to be tilted forward or backward, rotating about the front or the back edge of its lower base, or in other words, about an axis perpendicular to the paper in the side view. In this case it will readily be seen that the *side* view of the face B will not be changed, nor yet the apparent *breadth* of the front face A, the projection of which in the new position will still be bounded by the same vertical lines a, a.

In Fig. 77, therefore, supposing A and B to be the front and side views of the same prism as in the preceding figures, we first construct B', a copy of B; the edges a, b, c, d are still horizontal, and are seen in the new front view A' in their true length, and at the same distances above the new base-line $N'N'$ as in the side view: so that A' is readily constructed from A and B' by means of the T-square and triangles only. Next suppose a vertical reference plane, pp, to be passed through the edge a: then in the top view C' the other edges b, c, d will appear at distances from pp respectively equal to bt, cs, dr in the side view B'.

Instead of copying the original side view B, we might have drawn a new base-line, $N''N''$; then looking down perpendicularly toward this new horizontal plane, as shown by the arrow k, the edges a, b, etc., would have appeared of their true length, which is given in A: and thus the top view C'' is constructed, the process being exactly analogous to that adopted in Fig. 75. It is hardly necessary to point out that this result is precisely the same as if the top view C' had been laid out under the side view B', by projecting the edges a, b, etc., vertically downward, and measuring their lengths from A'.

126. Similarly, we may imagine the prism in Fig. 68 to be inclined to the right or the left, by turning about an axis perpendicular to the front plate of the glass case. The *front*

view will then evidently undergo no change in form or size, nor will the *thickness*, as seen in the side and top views. The construction of the drawing in this new position is so similar to

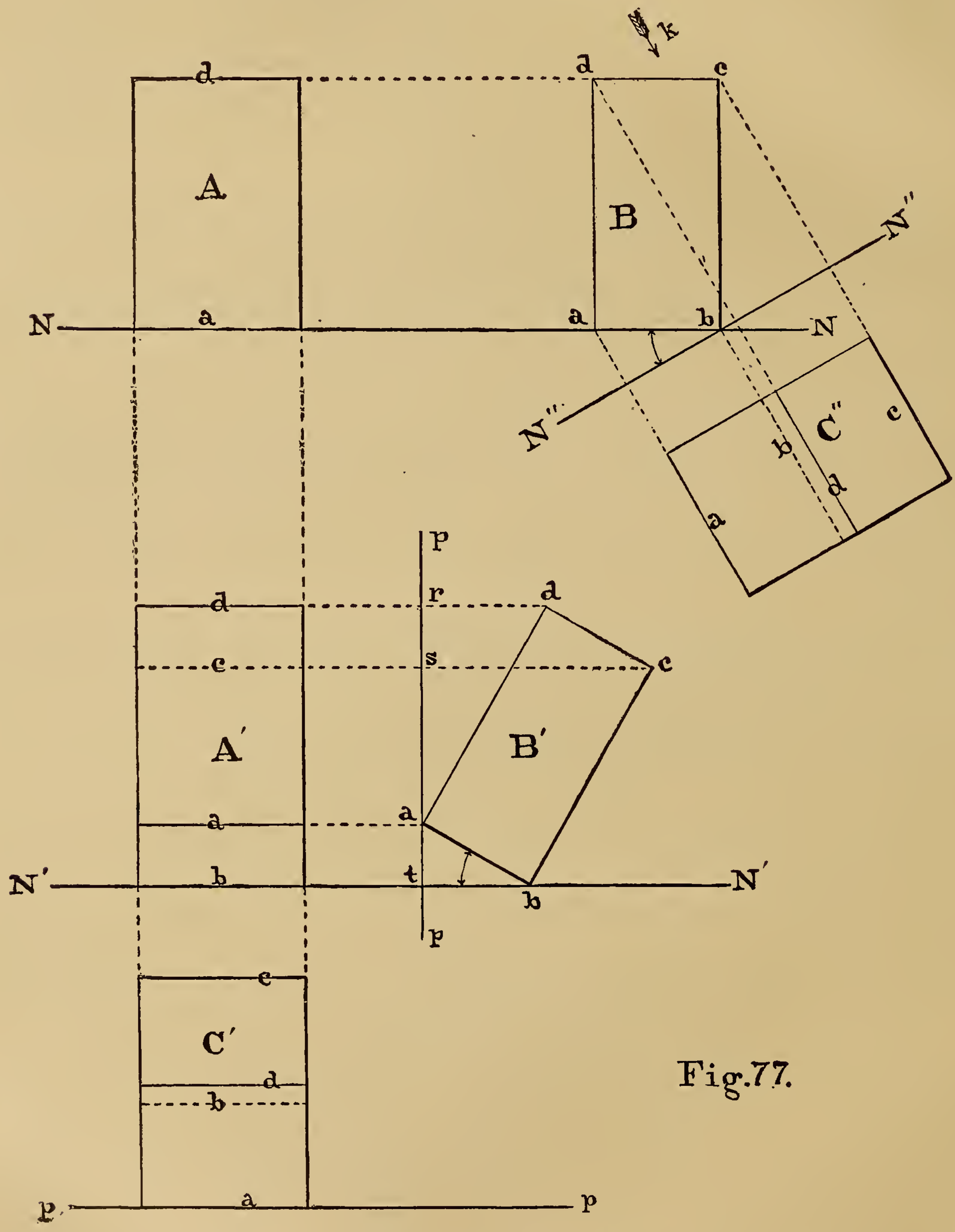


Fig. 77.

that in the preceding case that no further explanation is necessary; and indeed Fig. 77 illustrates it perfectly, if we regard *B* as a front view and *A* as a side view.

127. In each case, then, the prism is changed from its original position in Fig. 68, by

rotating it about one of its edges as an axis; in each case the view in the direction of that axis, and also all measurements parallel to it, remain unchanged. Clearly, the same would hold true, whatever the object represented, and the rectangular block was selected only on account of simplicity. But before going on to exercises of greater complexity, it is to be noted that no one of the changes thus far made is sufficient to meet all possible requirements; but by combining them the prism may be drawn in any position which can be assigned.

128. In Fig. 78 the same object is represented in the position described in (**126**). We may now regard this as the original position, and treat it in the same manner as before. In Fig. 79, for example, is shown the result of next turning it about a vertical axis; the top view, C', being therefore a copy of C in Fig. 78, but in a different position. Since the heights also remain unchanged, the new front view A' is constructed by projecting the points a, b, c, d, etc., vertically upward from C', with the triangle, and horizontally across from Fig. 78 with the T-square.

In the side view B', which is from the right, as indicated by the arrow k, the heights are also the same as in A'; and this view is constructed precisely as was the corresponding view in Fig. 76, by measuring the distance of each point from a vertical reference plane pp: which for convenience is passed through the extreme left-hand corner d; thus, for instance, in both C' and B' the distances as, ft, etc., are of the same magnitude.

129. It need hardly be remarked, that instead of copying the view C from Fig. 78, we might have adopted the course illustrated in Fig. 75. Thus, drawing a new base-line $N''N''$, project the various points a, b, c, etc., perpendicularly toward it: the height of each point above the new base-line is the same as that above the original base-line in the front and side views A and B, and is transferred from one of them by direct measurement. In this way the view A' in Fig. 78 is constructed: it corresponds to the view A' in Fig. 79, and would have been identical with it had the direction of the arrow m been determined with a view to such coincidence, as was done purposely in Figs. 75 and 76.

No new principle or mode of operation, then, has been introduced, but the object has been turned about two axes in succession. After the revolution about the second axis, it is to be observed that in two of the three views, the foreshortened representations of the faces of the prism are no longer rectangular, some of the right angles appearing obtuse and others acute. But each face is nevertheless represented by a **parallelogram,** in accordance with the deduction previously made, that all lines which in space are actually parallel will appear parallel in any projection; and if also equal in space, their projections will be equal.

This should be always kept in mind, not only because it serves to lessen the labor of construction, but because it is most important as a test of accuracy. And **the triangles should always be used as parallel rulers** in drawing the inclined sides; for since no measurements are to be made from these foreshortened views, it is preferable, if there is any error at all, that it should be in the magnitude, and not in the form.

130. We may now proceed to rotate the prism about the third axis, which is perpendicular to the paper in the side view. The form of this view is not changed by this rotation, and B'' in Fig. 80 is a copy, in a new position, of B' in Fig. 79. In making this copy, it will be found more accurate and more expeditious to construct first the rectangle $syxt$ within

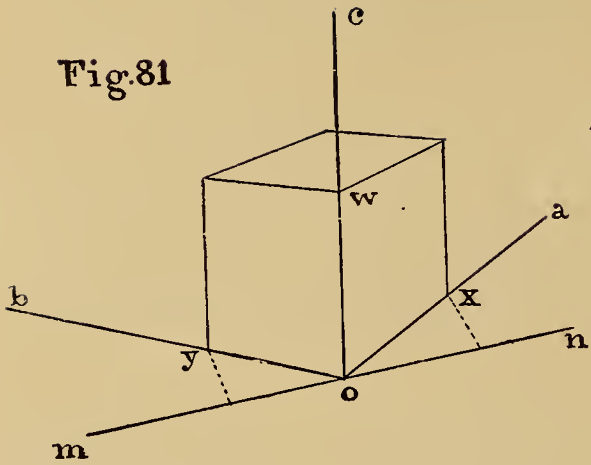


Fig. 81.

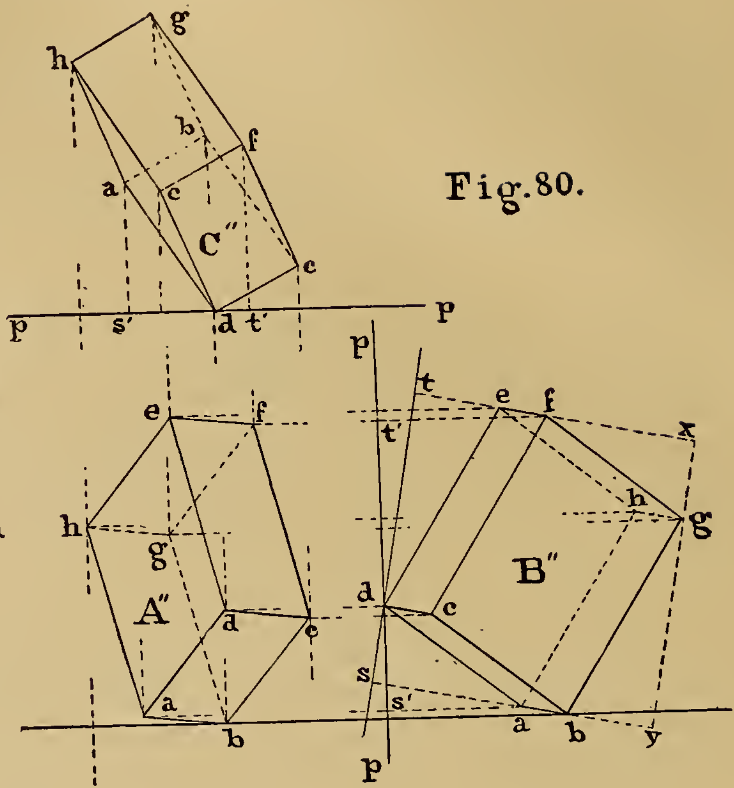


Fig. 80.

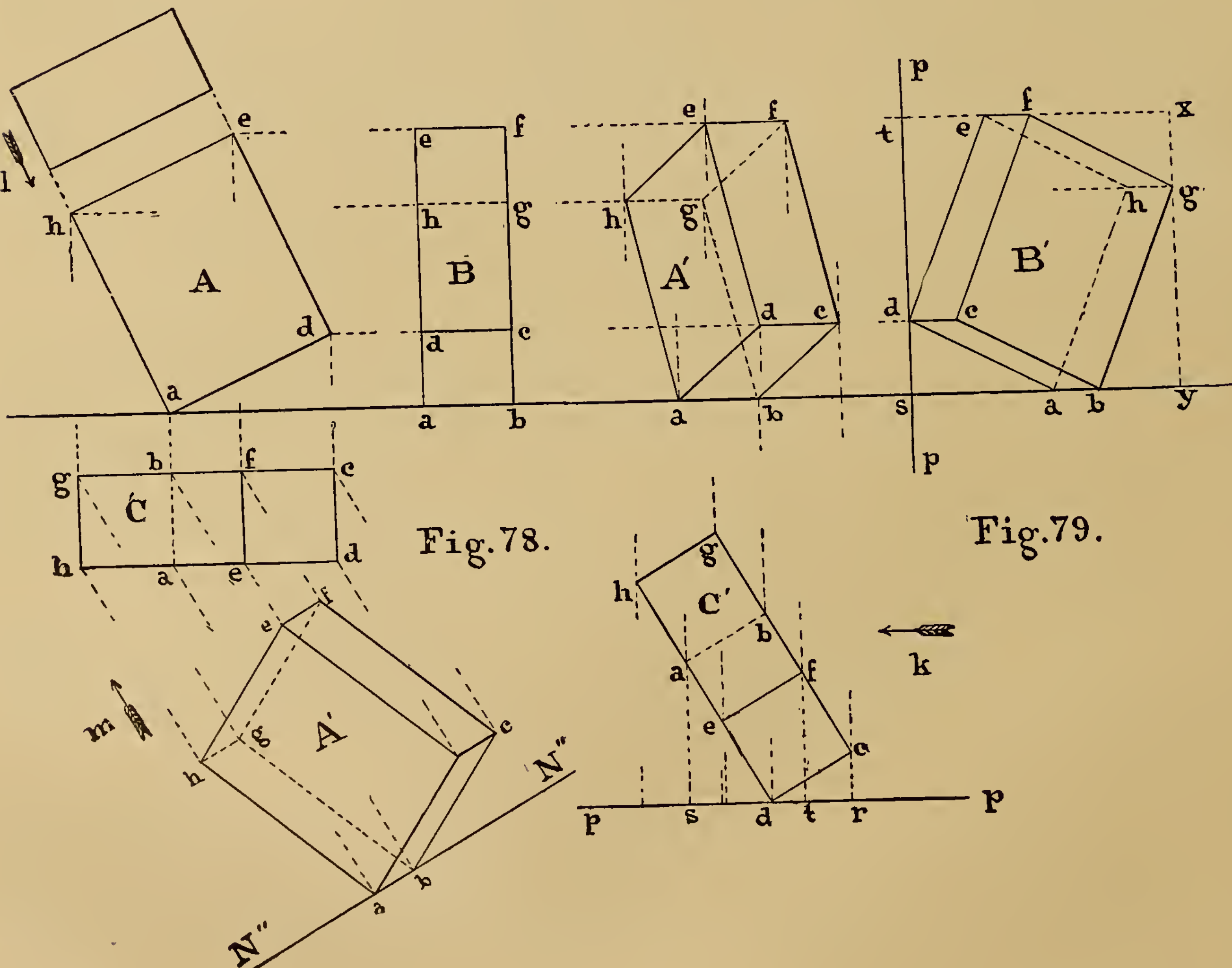


Fig. 78.

Fig. 79.

which B' is enclosed, and then to set off along its sides the distances *sd*, *sa*, *by*, etc., thus avoiding the direct construction of acute or obtuse angles.

The front view A'' is next laid out by projecting each point vertically upward from the view A' of Fig. 79, and horizontally across from the side view B''. In the top view C'' each point is of course vertically over its representation in A'', and its distance from any convenient reference plane *pp* is the same as in the side view: thus as', ft', etc., are equal in these views,—precisely as in Fig. 79.

As in the cases previously considered, the copying of the side view might have been avoided, and B' made to serve as a member of the group in the final position by drawing a new base-line, and transferring from A' the measurements of breadth for the new front view; but in this case with less advantage, if indeed with any, in respect to the saving of labor; for which reason the process has not been illustrated.

131. Of course, by rotating the prism about the axes in different order, or in different directions and through different angles, it may be placed in other positions than those shown in these diagrams: the student should now be able to construct these for himself, and will find it a profitable exercise to do so.

Since there are at present but three dimensions in space (and there is no immediate prospect of an increase in the number), the prism shown in Fig. 81 may by successive rotations about the three axes *oa*, *ob*, *oc* be placed in any position whatever.

The operations which have been explained, then, cover the whole ground, so far as this object is concerned: given at the outset its projections in the simplest position, we are now able to represent it correctly in the most complicated position, in which none of its sides appears of its true length, and none of its angles is shown in its actual magnitude.

In dealing with objects of any other form, the same principles hold true, and the same modes of procedure are employed. But there is a means of abbreviating the work when it is required to represent an object in such a position as that shown in Fig. 80; and in explaining this it may be as well to adhere to the simple prism for illustration.

132. The position alluded to was derived from the one shown in Fig. 68, by successive rotations about axes parallel to three adjacent edges of the prism. And the proposed abbreviation depends upon the fact that the successive rotations about any two of these are equivalent to a single rotation about another axis lying in the plane of those two, and passing through their intersection. This will be easily seen by the aid of Fig. 81, in which *oa*, *ob* lie in the horizontal plane of the base.

A rotation about *oa* will raise the point y above that plane; a subsequent rotation about *ob* will produce the same effect upon the point x, while the point o will remain fixed. It is clear that a single rotation about any horizontal line *mn* passing through o will raise both x and y above the plane, while o as before will remain stationary. The more nearly *mn* coincides with *ob*, the higher will x be raised by a rotation through a given angle, and the more nearly it coincides with *oa*, the higher will y be raised. By properly selecting the direction of *mn*, therefore, it follows that both x and y may be lifted to any given distances above the horizontal plane; this will of course determine the inclination to that plane of the new position of *ow*, which line will remain in a vertical plane perpendicular to *mn*. The

direction of mn and of the rotation about it being arbitrary, any possible position can be given to the prism by an additional rotation about the vertical axis oc.

133. It will often happen in practice that the inclination of the base will be given by requiring that the points x and y shall be higher than o by certain given distances; and the first thing is to determine the direction of mn which will effect this, which may be done as in Fig. 82. It is clear that the distances through which those points will rise are directly proportional to their distances, xd and yg, from the axis mn. Let these be, for example, in the ratio of three to five: divide oy into five equal parts, and set off ol equal to three of those parts; draw xl, then mon parallel to xl is the axis required. For, drawing xd and yg perpendicular to mn, and producing xl to k, we have

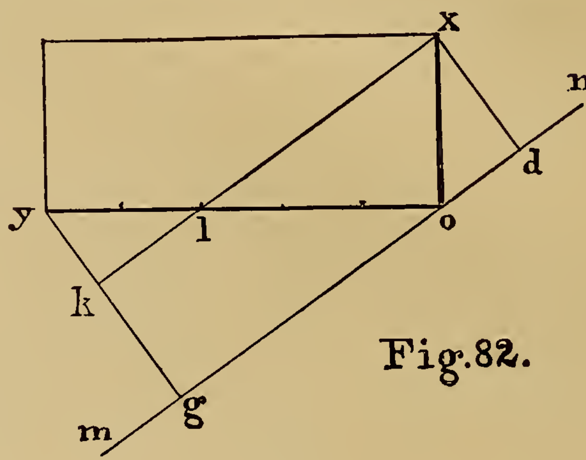


Fig. 82.

$$yg : kg \quad \text{or} \quad xd :: yo : lo.$$

134. Now let it be required to draw the projections of the prism shown in Fig. 83, after it has been turned first about mn (which will cause the vertical edge, represented by b in the top view C to appear in the direction bw perpendicular to mn), and subsequently around a vertical line at b, through the angle wbz.

The first step is to construct, as in Fig. 75, the view A', looking in the direction mn. Then, supposing that the height to which the corner c, for instance, is to be raised had been assigned: describe about c in this view A' an arc with radius equal to that height, and draw the base-line $N'N'$ through a, tangent to that arc.

Next construct a top view C'', looking down perpendicularly to the new horizontal plane thus indicated. In this view the edges ef, gh, etc., will appear parallel to $N'N'$, and their distances from any convenient vertical reference plane pp will be the same as in the original top view C. This new view C'' evidently represents the object with the required inclination to the horizontal plane: now make the angle $z''bv''$ equal to the angle zbv, shown in connection with the view C; then $v''b$ will be the direction in which we must look to make the front view in accordance with the assigned conditions.

135. Draw then a final base-line $N''N''$ perpendicular to $v''b$; project each point in C'' perpendicularly toward this, the horizontal plane, and make the height of each point above $N''N''$ equal to its perpendicular distance in A' above $N'N'$: this new front view A'' is the one required. From A'' and C'', a side view B'' is now readily made by the aid of a vertical reference plane $p'p'$, the distance of each point from this plane being the same in the side view as in the top view C''.

Now, holding the paper so that $N''N''$ is horizontal, the three projections are seen to represent correctly the prism in the position called for; and as there is but one intermediate view to make in passing from the simplest to the most complicated position, the labor of construction is reduced to a minimum.

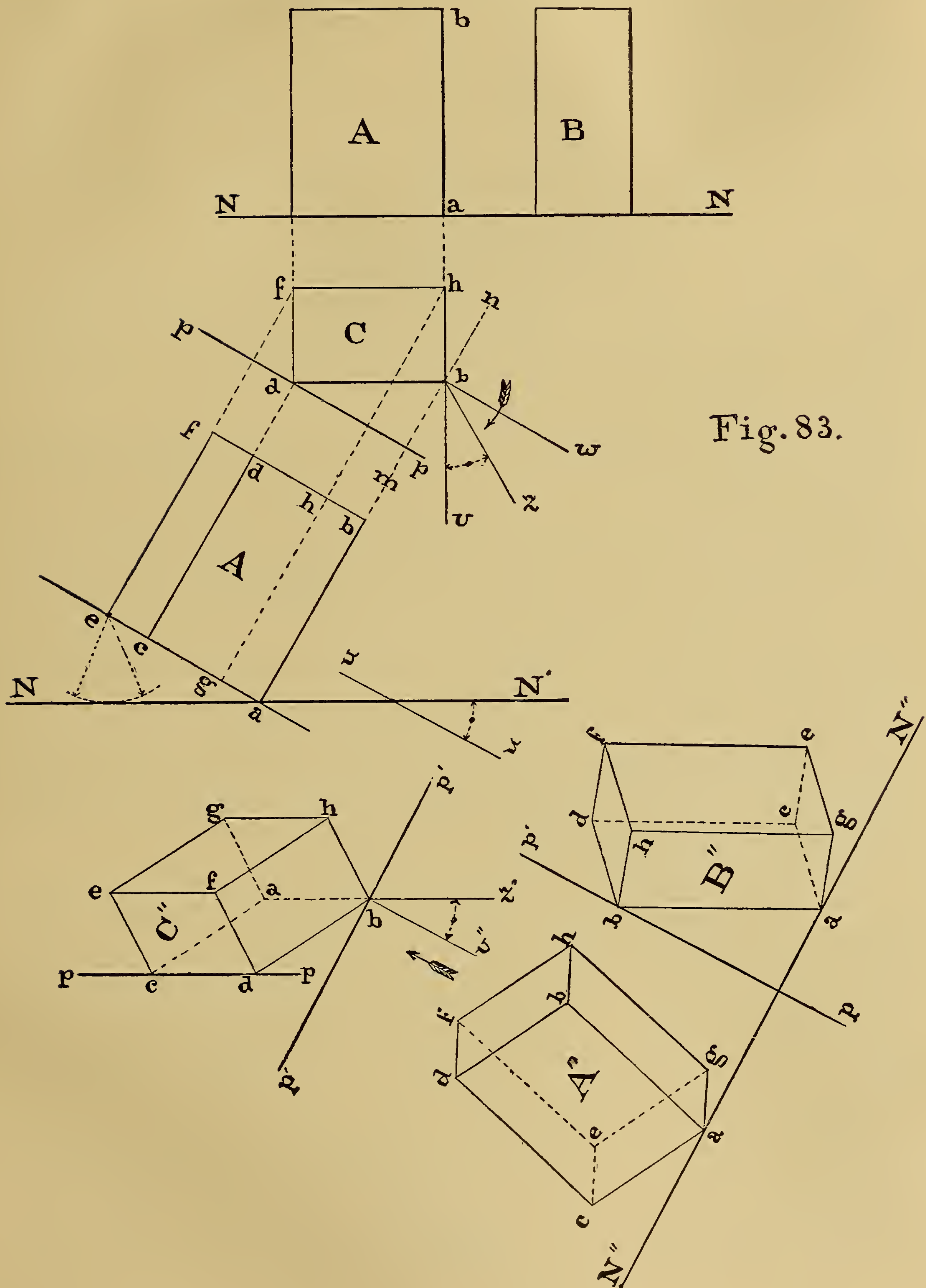


Fig. 83.

136. It will probably be most often the case that such views are required to be so placed upon the paper that the base-line shall be horizontal: usually, also, there would be no reason for keeping a record of the process of construction, and therefore no occasion for inking in the intermediate view A'. Nor is there any necessity for even pencilling it upon the paper; which may be avoided by constructing it upon a small piece of tracing-paper, on which should also be traced the original top view C, with the line *pp*. Next drawing a line *uu*, making with $N'N'$ an angle equal to *vbz*, the tracing-paper may be turned around and so adjusted as to make *uu* vertical; after which C'' is drawn at once upon the sheet, below the edge of the tracing-paper, which is removed after the other views have been constructed.

137. It should now be clear that this ultimate position of the object might have been defined by giving the angle which a line, originally vertical, shall make with the horizontal plane, and also the position of the vertical plane in which that line shall lie; for in Fig. 83 the former is determined by the first rotation about *mn*, and the latter by the second rotation about the vertical axis *ab* in the front view A.

Thus in Fig. 84 the vertical axis *oa* appears in the top view C as the point o: let it be required to draw the pyramid when this axis lies in the plane *ow*, and makes a given angle with the horizontal plane.

First construct the view A', looking perpendicularly toward the plane *ow*; in this view the base of the pyramid will appear as the straight line *bd* perpendicular to *oa*, and $N'N'$, making with *oa* the given angle *ocb*, will represent the horizontal plane. From A' and C the new top view C'' is constructed as in Fig. 83; make the angle $w''o''v''$ equal to the angle *wov* in the original top view C, then $N''N''$ perpendicular to $o''v''$ is the final base-line, from which are set up first the front view A'' by means of C'' and A', as in Fig. 83; then the view D'' from the left by means of A'' and C'', exactly as in the preceding cases.

139. Fig. 85 is a simple exercise, in which A, B, C are the front, end, and top views of a regular hexagonal pyramid, one of whose faces lies in the horizontal plane; D is a view looking perpendicularly at the base, which is thus shown in its true form and size,—an expedient often of great use; and C' is a perpendicular view of the upper face *cod* of the pyramid, thus showing the true lengths of the slant edges.

In this figure the circle circumscribing the base of the pyramid is drawn in the direct end view D; and this circle must, evidently, be represented in the foreshortened top view C by the curve which passes through the points a, b, c, etc. The base of the pyramid might have had twenty sides as well as only six, without introducing any new principle in projecting it; and the more numerous the points in the circumference of the circle, the more accurately will the foreshortened view of it be determined. But one or two further considerations in regard to drawing such projections of the circle are worthy of special note, since they are very frequently required in working plans of machinery.

140. In Fig. 86 the view A shows the edge of a circular disk, situated as was the base of the pyramid in Fig. 85; and *ab* in this view also represents the side of the circumscribing square, which is drawn in the view D. In the top view C this square will be foreshortened into the rectangle shown, and the circle will appear as an ellipse, of which *xx* and *yy* are the axes. Now constructing the view A'', looking horizontally at C in the direction of the arrow v, the circumscribing square will appear as a parallelogram *abcd*, to the sides of which *xx* and

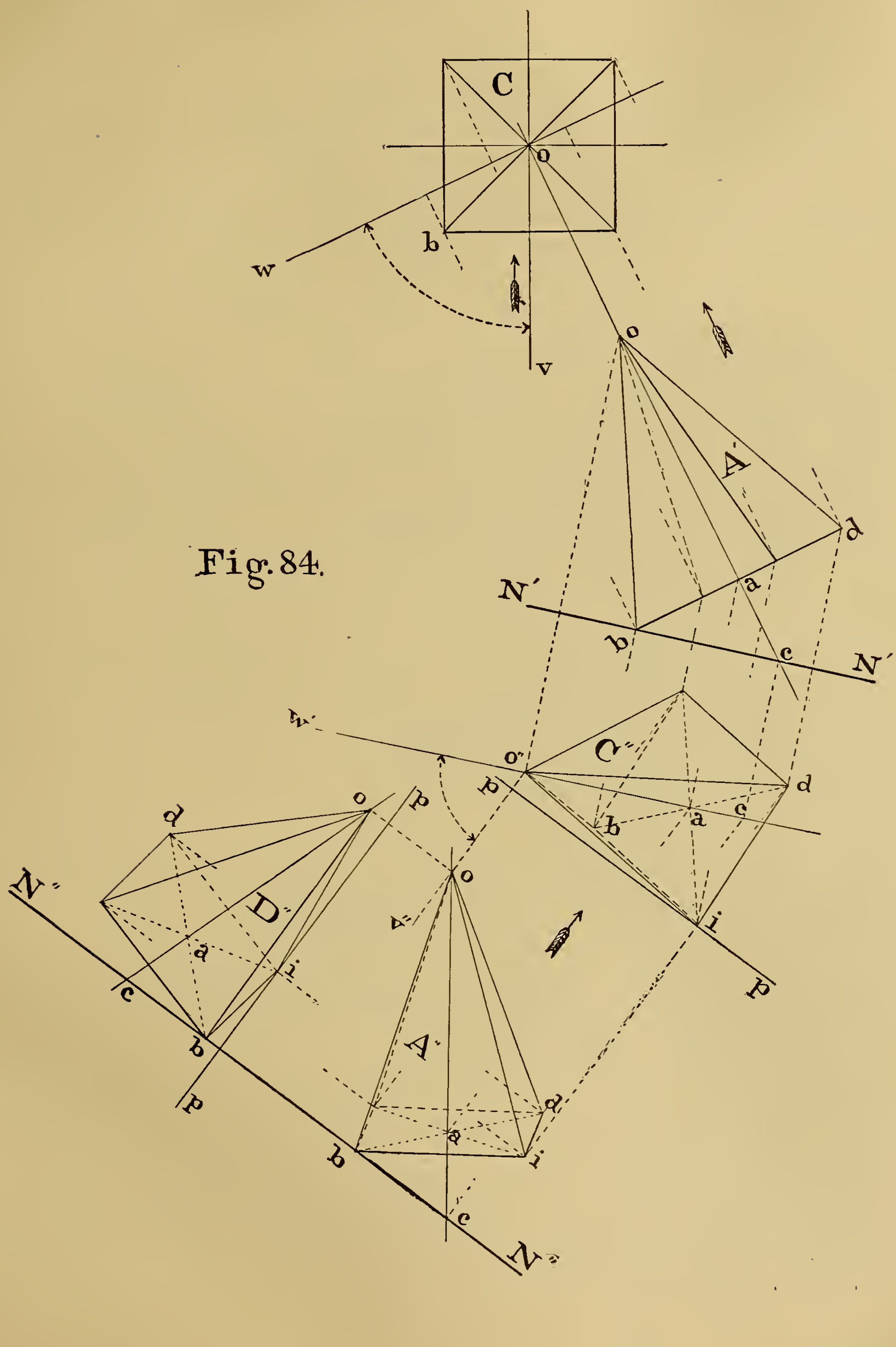


Fig. 84.

yy are parallel: these lines are conjugate diameters, but not the axes, of the inscribed ellipse.

In order to find the axes, we may proceed as follows: Draw a visual ray, that is, a parallel to the arrow v, through the centre o in the view C. This ray being horizontal, a plane perpendicular to it will be vertical, and will cut the plane of the circle in the line hg, which will be a diameter, because it passes through o, and will in the view A'' appear of its true length.

Producing hg in the view C to cut ad and bc in m and n, project these points perpendicularly to ad and bc in the view A'', draw mon in that view, and set off oh and og equal to

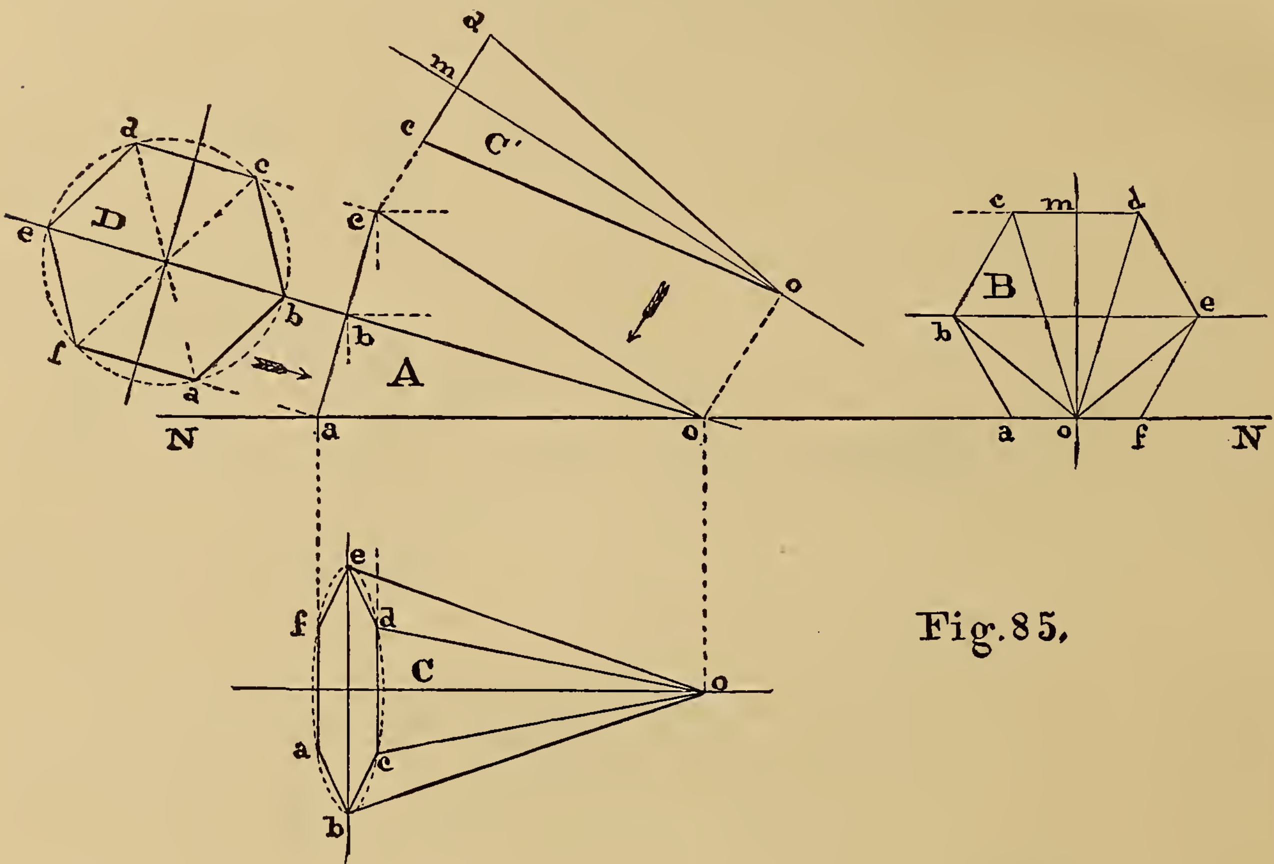


Fig. 85.

the true radius of the circle, given in the view D: then hg is the major axis, since it appears of its true length, while every other diameter is foreshortened.

141. Since the minor axis is perpendicular to the major, draw in the view A'' a perpendicular to mn, cutting ab and cd in k and i; project these points back to the view C, in which draw ik cutting the ellipse in p and q, which latter points being projected again to ik in A'', will determine the length of the minor axis required.

This, however, depends upon the perfection of the ellipse in the view C; and if the greatest accuracy is desired, the following method is preferable: Observing that lines parallel to xx are not foreshortened at all in the view C, we may transfer the measurements ym, yn directly to the view D, and draw mn in that view, also ik perpendicular to it, thus locating p and q in the original position of the circle. These points are then projected to the edge view A, thus determining their distances above the horizontal plane NN. Finally, in the view A'' draw parallels to $N''N''$, at the distances above it just ascertained in A; which will cut ik at the required extremities of the minor axis.

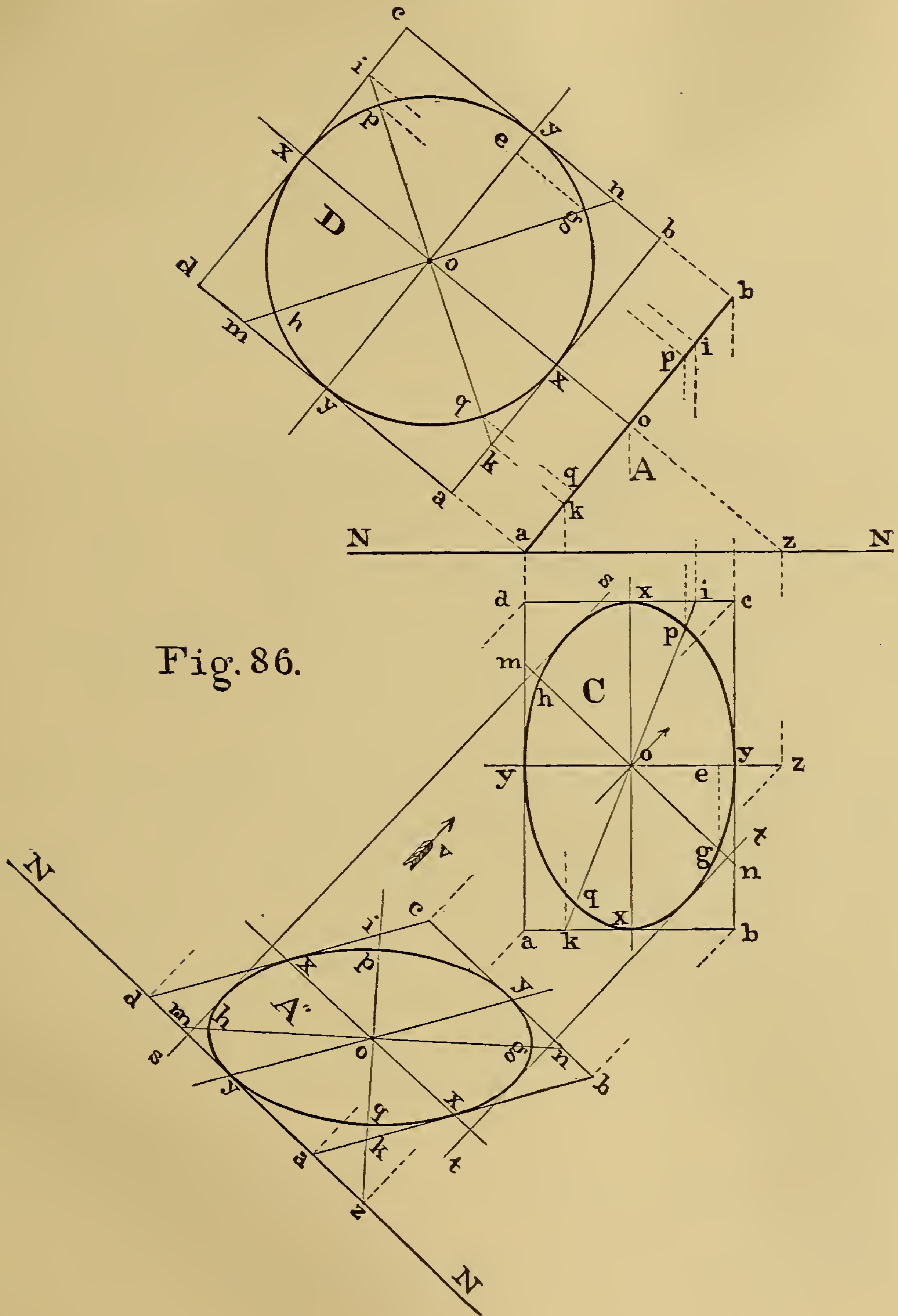


Fig. 86.

142. These operations are, evidently, outside of the range of projection pure and simple, for they depend upon the mathematical considerations that any projection of a circle is an ellipse, and that any two diameters of the circle which are at right angles to each other will in the projection appear as conjugate diameters of the ellipse. Had the curves been determined simply by projecting a number of points, of course the same result should follow; but the shortest way is always the easiest, and it is perfectly proper to save time by availing ourselves of those known properties for finding the axes, and then to construct the ellipse by any method that is most convenient. And sometimes it is quite important to draw the curves with precision,—as for example in making inclined projections of toothed wheels.

In many cases, however, no such minute accuracy is necessary,—as for instance in representing an inclined cylinder; and in Fig. 87 is shown a shorter process well adapted to such purposes.

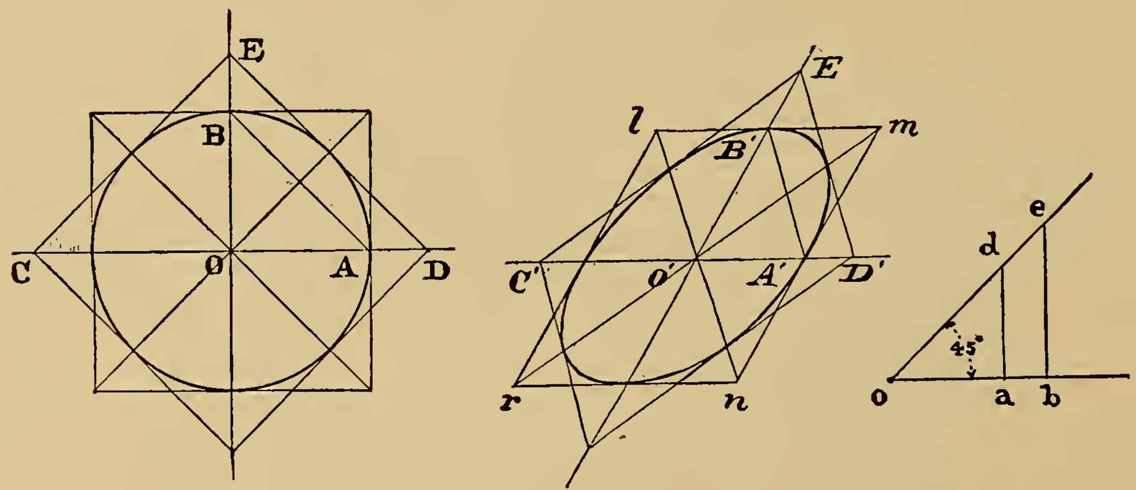


FIG. 87.

143. The circle is shown as circumscribed by two squares, the diagonals of each being parallel to the sides of the other: it is clear then that OA and OD, which in the projection appear as $O'A'$ and $O'D'$, will be foreshortened in the same proportion, and so will OB and OE. This fact renders it unnecessary to draw the second square in the original or direct view of the circle. For, having found the projections of one, *lmnr*, and of its centre-lines, as in Fig. 86, the lengths of the diagonals of the other may be ascertained by a very simple process shown in the diagram at the right: draw a horizontal line, and another making an angle of 45° with it; from the vertex set off *oa*, *ob*, respectively equal to $O'A'$ and $O'B'$; at *a* and *b* erect vertical lines cutting the inclined one in *d* and *e*. Set off $O'D' = od$, and $O'E' = oe$, and draw the sides of the second parallelogram parallel to the diagonals *mr*, *ln*, as shown. We have thus eight points in the required ellipse, and what is of still more service, the tangents at those points.

It may perhaps be added, that short portions only of the lines made use of in this illustration need be actually drawn in practice; with due attention to this precaution, this method is both neat and expeditious.

144. Application of the foregoing may be made in constructing the projections of the cylinder given in Fig. 88, of which no detailed explanation is needed.

But in whatever position the cylinder is placed, it is possible to conceive two planes

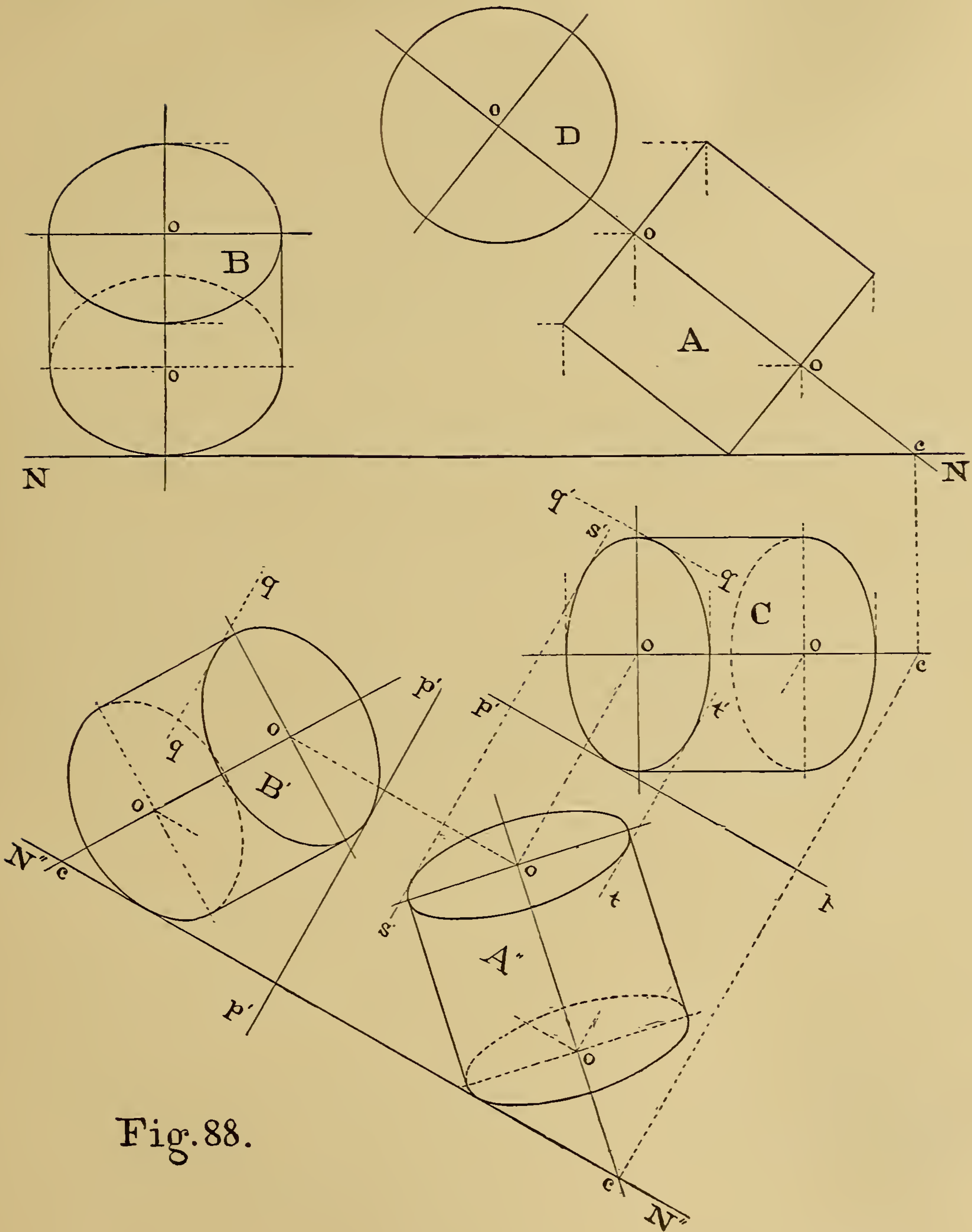


Fig. 88.

tangent to it on opposite sides, and parallel to the direction in which we are looking: these planes will touch it along two lines parallel to the axis, and the least distance between them will be equal to the diameter of the cylinder.

From this it follows, that in any projection the apparent diameter will be equal to the real diameter; and also that if in any view, as for instance A'' in Fig. 88, we first determine oo, the projection of the axis, then a line perpendicular to that projection at either extremity, and equal to the actual diameter of the cylinder, will be the major axis of the ellipse representing the corresponding base.

And it is to be noted also that in Figs. 86 and 88 the ellipses in views C and A'' have on opposite sides the common tangents ss, tt, perpendicular to $N''N''$; which fact, if the first ellipse is correctly laid out, is of service in constructing the second, even if the point of contact be not determined—which indeed is practically of no consequence whatever.

CHAPTER V.

OF THE HELIX, AND ITS APPLICATION IN THE DRAWING OF SCREWS.

145. **The helix** is a curve traced upon the surface of a cylinder by a point which moves at a uniform rate around the circumference, and at the same time travels uniformly in a direction parallel to the axis. But though both motions are uniform, their rates are independent of each other; while going once around the cylinder the point may advance in the direction of the axis to any distance, great or small, at pleasure, and this distance between the successive coils is called the **pitch**. The directions of the two motions are also independent of each other, and thus the helix may be either **right-handed** or **left-handed**: to use a homely illustration, there are two directions in which a string may be wound around a stick.

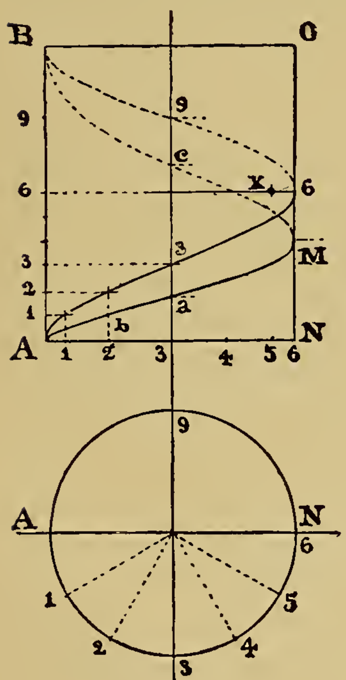


FIG. 89.

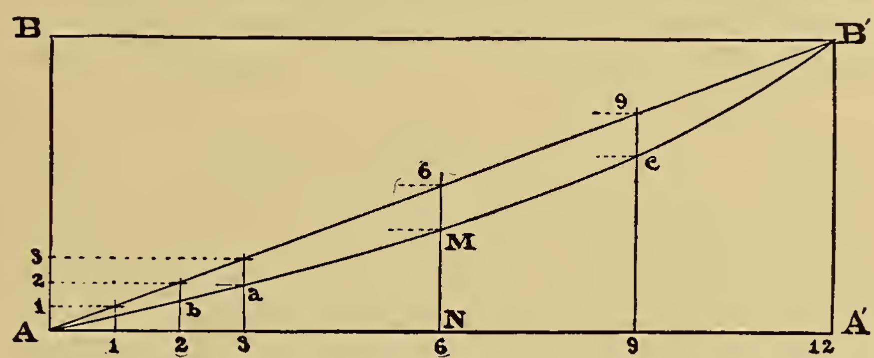


FIG. 90.

In Fig. 89 are given a side view and a top view of a vertical cylinder. In the top view, beginning at A, divide the circumference into equal parts by the points 1, 2, 3, etc., and project these points up to the lower base of the side view; where of course the divisions, being unequally foreshortened, will not appear equal.

In the side view set off along AB equal spaces, numbering them also 1, 2, 3, etc., from A. Through each of these points on AB draw a horizontal line, and through the divisions on AN draw vertical lines: the intersections of those correspondingly numbered will be points upon the helix $A369B$. The portion $A36$, lying on the front side of the

cylinder, is visible, and shown in a full line; the portion $69B$ is hidden, lying on the back of the cylinder, and therefore shown in a dotted line.

146. It is easy to see that the visible portion $A36$ is divided at the point 3 into two equal and similar parts, $A3$ being concave toward AN, while 36 is convex toward that line. The curvature is most rapid at the points A and 6, diminishing toward the point 3, which is the point of **contrary flexure.** The radius of curvature being infinite at this point, the curvature of the helix for some distance on either side of it will be almost imperceptible, and this part of the line is practically best drawn with a straight-edge. And the same is true with regard to the concealed portion $69B$.

It is equally apparent that the visible part is exactly similar to the invisible part; the horizontal line 66 is a line of symmetry, and the curve $69B$ above it is a reproduction of $63A$ below it, in a reversed position.

147. Moreover, **the projection of the helix is tangent to the visible contour of the cylinder**: thus, AB is tangent to the curve at A and B, and NO is tangent to it at the point 6, which is the vertex of the portion 369, symmetrical about the horizontal 66.

For this reason it is advisable always to use the bow-pen for drawing a small portion of the curve on each side of the vertex, finding by trial and error the proper radius and the centre x upon the line of symmetry 66; the same radius being of course used wherever applicable at other points, as at A and B in this figure.

Since the curvature changes most rapidly near these vertices, care should be taken to find points near each other in these parts, in order to locate the centre x with a reasonable degree of precision; which is absolutely essential to a successful representation of the helix, an error at the vertex being more conspicuous than in any other portion of the curve.

148. Suppose the cylinder to be made of a thin sheet of metal, and cut lengthwise along the line AB: it may then be unrolled into a flat sheet, shown in Fig. 90, of which the breadth AB is equal to the length of the cylinder, and the length AA' is equal to the circumference. This is called the **development** of the cylinder; and since the unrolling involves no distortion, extension, or compression in any part of the metal sheet, the equidistant points 1, 2, 3, etc., on the circumference in Fig. 89 will appear as equidistant points on AA' in Fig. 90; and the same is true of the divisions on AB. Therefore, drawing horizontals through the latter and verticals through the former, the intersections of the correspondingly numbered ones must be points on the development of the helix. From the constant ratio between the bases and the altitudes of the triangles thus formed, it follows that this development must be a right line: which in this case is a diagonal AB' of the rectangle, because the pitch of the helix is equal to the length of the cylinder.

149. It is quite plain that this operation might be reversed; were a sheet of thin paper cut into the form of the triangle $AA'B'$, and rolled around a wooden cylinder of the given dimensions, its hypothenuse would coincide with the original helix, the vertical lines upon it also coinciding with those first drawn upon the cylinder.

Now suppose the upper edge of the paper cut in the form of the curve AMB': it is clear that this edge when wrapped round the cylinder would *not* form a true helix. But we can easily determine the projection of the curve which it would form, since the vertical lines on the paper would go back to their original places on the cylinder, and their lengths are given

in the development. In constructing the drawing, then, we have only to project the points a, b, M, c, etc., in which these verticals cut the curve in Fig. 90, horizontally back to the correspondingly numbered verticals in Fig. 89.

It follows from the above that the shortest line which can be drawn on the surface of a cylinder, between two given points which lie neither upon the same circumference nor upon the same right line, is a helical arc. For on the development these points will appear at their true distance from each other; and the right line joining them, being by the conditions neither horizontal nor vertical, will become a helix when the developed sheet is re-formed into a cylinder.

150. The helix is sometimes called a *linear screw*, and may be traced by the point of a cutting tool moving endwise at a uniform rate along a cylinder revolving in a lathe, and just in contact with its surface. If the tool be fed farther in toward the axis, a helical groove will be cut, of a form depending on the shape of the cutting part of the tool; and if the pitch be great enough, a helical projecting ridge will be left, and this is called a screw-thread. We say if the pitch be great enough; for if the longitudinal travel be very slow, it is easy to see that this thread will be entirely cut away, and the operation will result in merely turning down the original cylinder to a smaller diameter. By varying the shape of the tool, then, screws of various forms may be produced, of which the most common are the square-threaded and the V-threaded.

THE SQUARE-THREADED SCREW.

151. A clear idea of the square-threaded screw may be formed by imagining a square rod of lead to be coiled around a cylindrical core, like a string around a stick, leaving a space, as wide as the leaden bar itself, between the adjacent coils. Thus in Fig. 91 $acif$ is the section of this bar; after making a half turn it appears in the position $cglm$, and at the end of a whole turn in the position $dkon$; since the groove is to be as wide as the thread, $fd = af$, and $ik = ci$. Also, af, cm, dn, jz will lie upon the surface of a larger cylinder, in which the helical groove might be cut as before explained.

The line ae is perpendicular to the axis, which it would cut at v, and if still farther prolonged it would cut the opposite sides of the two cylinders at u and w. Now ad, or its equal fn (being the breadth of the thread plus that of the groove), is the pitch of the screw; in going from a to d, the point a describes a helix $abcd$, lying on the surface of the outer cylinder, and in going from e to k, the point e describes a helix $ebgk$, lying on the surface of the smaller cylinder: similarly, f and i describe the helices $fhmn$, $ihlo$, and all the other curves, for successive threads, are simply repetitions of these.

152. In order to make the drawing neatly, as few lines as possible should be pencilled in, and very few are in fact necessary. Having lightly drawn the outlines of the outer and inner cylinders, and the axis, an indefinite vertical line through a determines the points e, v, u, w; then, observing that $vb = \frac{1}{4}$ pitch, and $wc = \frac{1}{2}$ pitch, the vertices of the helices, as f, d, n, l, m, j, etc., as well as the points h, p, q, etc., on the axis, may at once be set off by means of the scale; and very short vertical lines only need be drawn with the triangles through the vertices, on which are located the centres of the small arcs to be drawn with the bows. Let

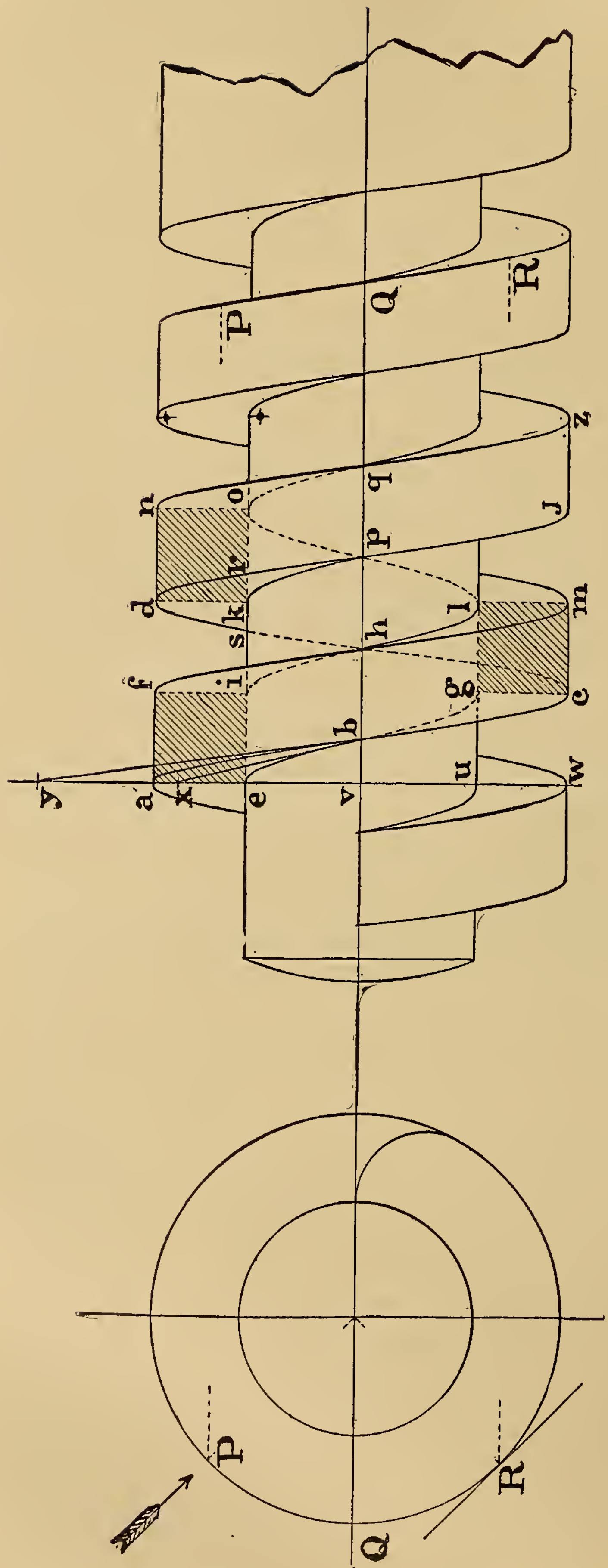


Fig. 91.

now the visible portions of the helices for one half of a thread, as *sdrp*, *nq*, *kp*, be carefully constructed and traced on a piece of tracing-paper, on which are also to be drawn the axis and the vertical lines *dk*, *no*. Turn the tracing-paper over and trace the lines carefully with a pencil of medium hardness, on the opposite side, then turn it over again to its original position; by sliding it along and adjusting it to the positions of the successive threads, the lines on its back may be transferred to the paper below by going over them carefully, with a firm pressure, with a pencil. Having thus made impressions or transfers for the upper halves of the threads, turn the tracing-paper end for end, and repeat the operation for the lower halves.

153. In the construction of the curves particular attention should be paid to the fact that the visible portion *ds* on the remote side is in the projection precisely similar to the corresponding portion *dr* on the nearer side, *dk* being the line of symmetry, so that $ks = kr$; and also to the fact that *kp* is not tangent to *drpj*, but intersects it at *p*, at an angle which may be determined as follows:

The helical arc *ab* embraces one fourth the circumference of the cylinder, and the corresponding linear advance is *vb*, one fourth the pitch. On *va* produced set off *vy*, equal to the quarter circumference, then *by* is tangent to the helix at *b*; and the angle *vyb* is the measure of the **obliquity** of the curve. With the same pitch, the obliquity will be the

greater the smaller the cylinder: set off then vx equal to the quarter circumference of the core, then bx is tangent to the inner helix, and vxb is its obliquity.

If, in inking in, small portions of the helices near the axes are drawn with the straight-edge, these tangents should always be constructed and the triangles set so as to make those straight portions parallel to them.

154. When a very long screw has to be drawn, it will facilitate matters and save time to make a template of a thin slip of white holly, cut to the exact form of the helical curve. The centre-line should be drawn upon this, and it is used in connection with the T-square as shown in Fig. 92; it is necessary only to slide it along to bring the point o successively to the positions b, h, p, etc., in Fig. 91. In making such a template, no attempt should be made to carry the helical curve farther than the points of contact with the small circular arcs at the vertices, as at f, m, Fig. 91, for neither pencil nor ruling pen can be successfully used to draw such sharp curves by any template, concave or convex; beyond those points, the outline should be straight, and tangent to the helix. Such small portions of the inner helices are

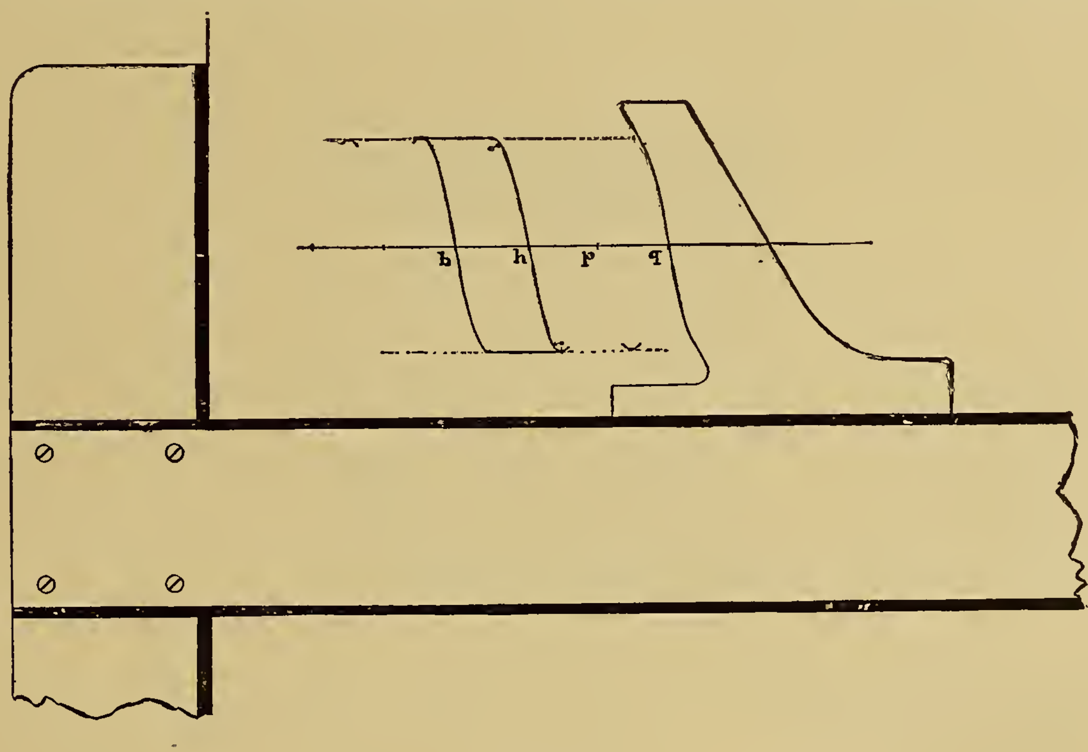


FIG. 92.

seen in the drawing, that it is seldom or never worth while to make a template for drawing them.

Should proper wood for the purpose not be at hand, it is well to know that such curved rulers can be made of heavy card-board or Bristol-board to good advantage: this should receive on each side one or two coats of shellac varnish, and when well dried it will then be found easy to produce a very satisfactory drawing edge by means of fine files and sand-paper.

155. The effectiveness of the drawing will be much enhanced by the proper use of shadow-lines. The arrow shown in the end view at the left of Fig. 91, being drawn toward the centre, is normal to the surface at P, at which point the light is most concentrated; the

corresponding point in the side view is therefore the one at which the shadow-line should be heaviest; from P to Q it may be substantially uniform, but from Q to R it should be gradually diminished in thickness, merging at the latter point into the unshaded outline. Undeniably, a shadow is cast by the helix up to the very highest visible point; but the surface above P is receding, and the shadow-line is therefore tapered off rapidly above that point, in accordance with the general principle that the heavier the shadow-line, the more pronounced is the effect of relief.

156. If we suppose an iron screw to be put into a mould as a core, and type-metal to be cast around it, it will be seen that this metal will exactly fill the groove. The surface of the nut thus formed, then, is identical with the surface of the screw: if now, without unscrewing the iron core, we plane off one half of the whole, down to the axis, it is evident that the outline of this longitudinal section of the nut will coincide precisely with that of the section of the screw. Consequently, if the remaining half of the screw be taken out, the interior of the nut will exhibit lines which, from the manner in which it was made, must be identical with those portions of the helices lying on the farther side of the screw, and therefore not visible in Fig. 91. Such a section of the nut is shown in Fig. 93, and requires no farther explanation, except

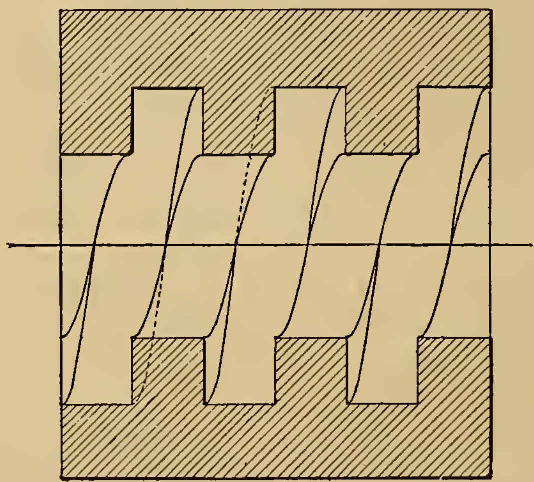


FIG. 93.

to call attention to the circumstance that all shadow-lines except those lying in the plane of the section are omitted, since all the edges which would cast shadows lie upon *receding* surfaces.

157. The screw shown in Fig. 91 is a **single-threaded** one; now if we wind the same bar around the same core, but double the pitch, the groove will be three times as wide as the thread. In the middle of this groove we may wind another bar of the same section, thus forming a **double-threaded** screw. Or, what comes to the same thing, if we cut in the lathe a groove of the same size as that shown in Fig. 91, but of twice the pitch, there will be left a thread three times as wide, in which another similar groove may be cut. A double-threaded screw, with a section of its nut, is shown in Fig. 94.

Clearly, the groove may be made deeper or shallower at pleasure, and wider or narrower than the thread; also, by trebling the pitch we may add another thread of the same size, and

so on indefinitely. But unless the diameter is increased, the obliquity of the helices will become greater with each additional thread, until at last it would be impossible to turn the screw in the nut if the latter were fixed.

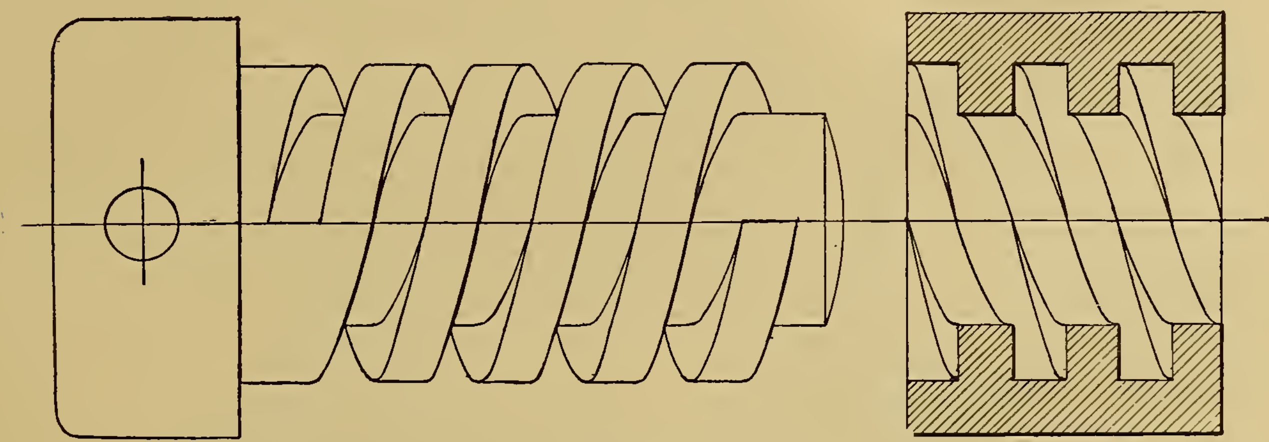


FIG. 94.

Examples of such multiple-threaded "quick" screws are found in the so-called "Archimedean" drills, which are rotated by sliding the nut endlong. None of these variations, however, involve anything in regard to the making of the drawings beyond what has been explained.

158. The operation of winding a rod or bar around a cylindrical core, which we have used as illustrating the formation of a screw-thread, is of course actually performed in the making of helical springs. The "wire" of which these are made is usually either round or square in transverse section, although other forms are occasionally employed.

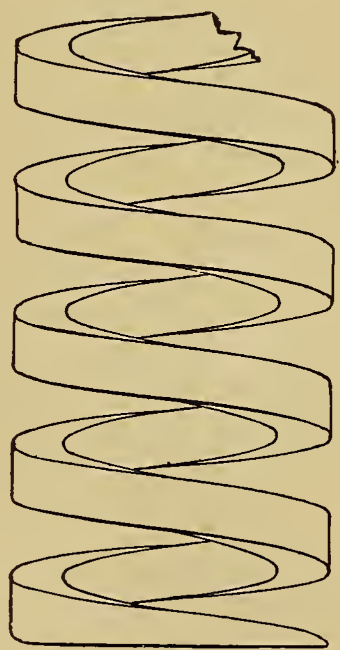


FIG. 95.

The appearance of the spring, after withdrawing the core around which a square bar has been wound, is shown in Fig. 95; it requires no special explanation, the visible lines being portions of the helices into which the edges of the bar have been twisted, and short parts of the rectilinear elements of the outer cylinder.

But if a round bar be thus coiled into a helical spring, its axis will be twisted into a helix lying on the surface of a cylinder, whose diameter is that of the core plus that of the bar. Now suppose a fine wire to be coiled into the form of this helix, and a hole of the same size and curvature as this wire to pass through the centre of a sphere of the same diameter as the bar. Since the curvature of the helix is everywhere the same, the ball could slide along the wire, and in every position would just fit a similarly coiled tube of the same diameter.

Having drawn the central helix, then, describe a number of circles, with centres on this helix, and a radius equal to that of the bar; the envelope, or curve tangent to all these circles

will be the visible contour of the coiled rod, as shown in Fig. 96. A moment's study will show that the line *ab* of this contour cannot extend beyond the point *i*, at which it is tangent to the outline of the sphere whose centre is *c*, the vertex of the central helix *ocp*; and in like manner the line *de* must disappear at *l*, where it is tangent to the outline of the sphere whose centre is *o*; and similarly in regard to the other coils.

The operation just described is the most expeditious as well as the most accurate method of determining the correct visible contour of the helical coil, and is to be employed when the object is of considerable dimensions, as in the case of a very large spring, or the worm of a still.

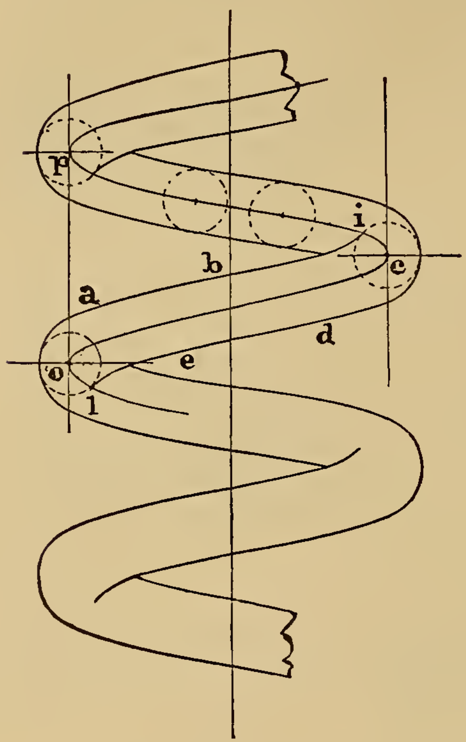


Fig. 96.

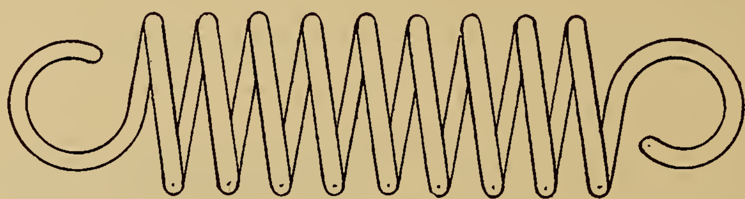


Fig. 97.

But springs are seldom so large as to render this labor necessary: under ordinary circumstances, the mode of representation shown in Fig. 97 answers all purposes, care being taken to locate correctly the centres of the small circles, to which straight parallel tangents are drawn; the curvature of the helix being slight, may be neglected.

THE V-THREADED SCREW.

159. This may be formed by winding a flexible bar of triangular section, base downward, upon a cylindrical core, as shown in Fig. 98. The pitch is here made equal to the base of the triangle, so that the sections of adjacent coils, as *aef*, *fgh*, just touch each other at the root of the groove left between them, where they have the common point *f*: this forms what is called the **full sharp** single-threaded screw, which of course is actually made in practice by cutting the triangular groove with a sharp-pointed tool.

Now in winding the flexible bar it is at once seen that the outer edge *a* is twisted into a helix *alq* lying on the outer cylinder, whose outline is *avg*; and the lower edge *e* into a helix lying on the surface of the core.

In like manner the intermediate points *b*, *c*, *d* of the side *ae* of the section will describe helices lying on intermediate cylinders: these will have different obliquities, because they have the same pitch, *ag* or *ef*. Were these actually engraved upon the surface of the screw, it is evident that they would be visible on the nearest side, up to and beyond the points *b*, *c*, *d*, but would finally disappear. Since they lie *upon* the surface, then, and do not go outside of it, the **visible contour** must be bounded by a line *ik*, **tangent to all the helices**: the line *ae*, on this side of the thread, is still visible; but it is not the outline, and therefore is not shown in the finished drawing.

160. Similarly, the visible contour of the right-hand side of the thread will be a line *lm*, tangent to all the helices described by the points on the line *af*. But since it *is* the visible contour, those helices will disappear from view at the points of contact with *lm*; the outline of the section *af* being thus concealed by an intervening portion of the thread.

Obviously, the same reasoning and the same process would enable us to determine the extreme outline of a thread whose section was of any other form, symmetrical or otherwise.

And it is equally clear that when, as is usually the case, the section *is* symmetrical with regard to a line, as *aw*, perpendicular to the axis, the contour-lines *ik* and *lm* will be similar, and also symmetrical in respect to the same line.

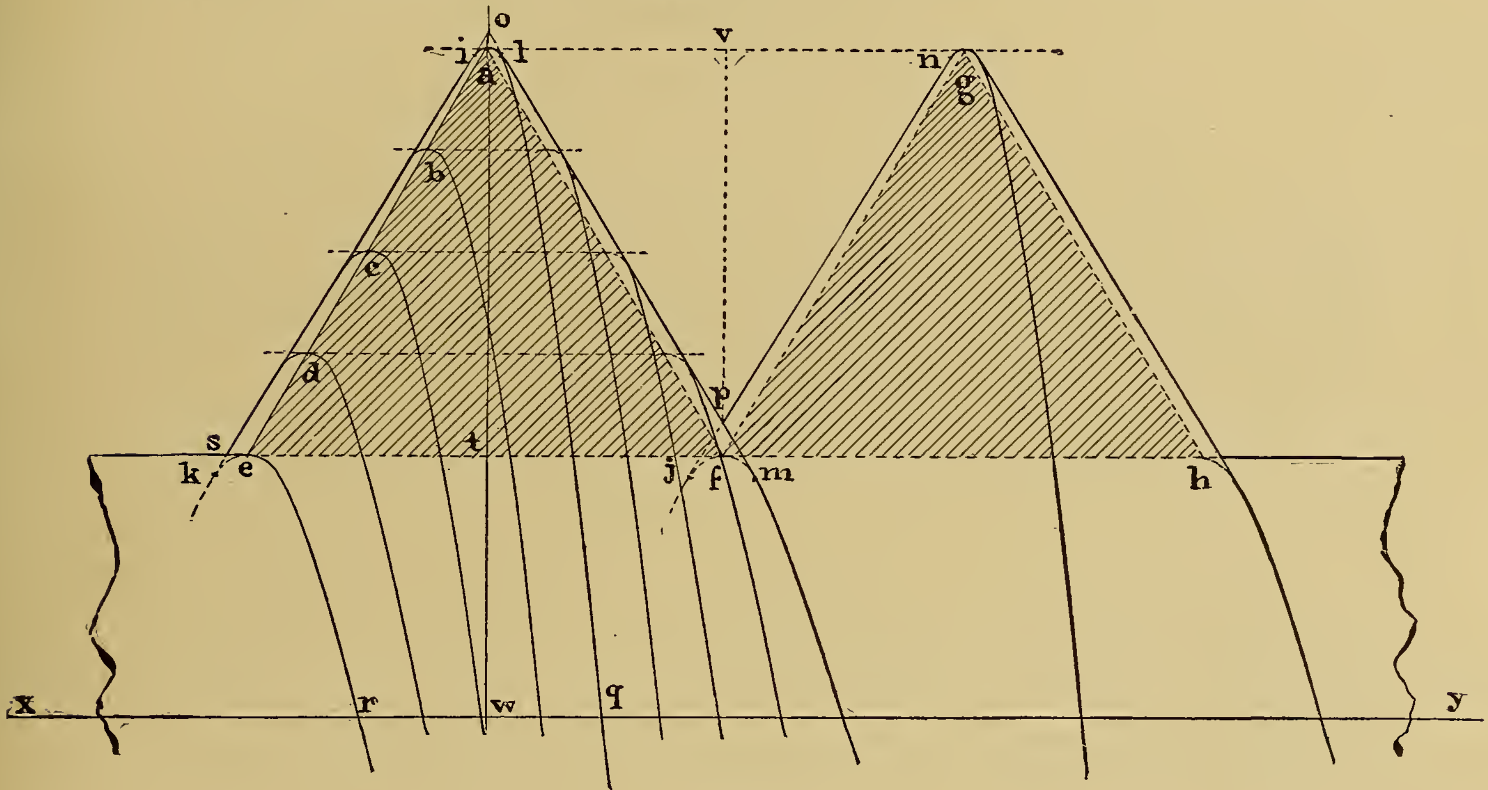


FIG. 98.

In the figure the triangle *aef* is equilateral, because in the U. S. standard screws the sides of the threads and grooves are inclined to each other at an angle of 60°. This being so, it is to be observed that the contour-lines are in reality slightly concave; but with the usual proportions between the size of the thread and the diameter of the core (practically determined by considerations of strength), the curvature is too slight to be appreciable.

161. It suffices, therefore, to draw only the outer and the inner helices, *aq* and *er*, carrying them well beyond the vertices *a* and *e*, and to draw a straight line tangent to these two. This tangent should be prolonged to cut the vertical line through *a* in the point *o*: then *lm* will also pass through *o*, and the angle *wom* will be equal to the angle *wok*; also, be it noted, each of these angles will be greater than *wae*.

The contour-lines will be tangent to the outer helix at two points *i* and *l*, on opposite sides of the vertex and equally distant from it; and this small arc is best drawn with the bow-pen. The thread, then, though actually sharp at the top, does not appear so in a correct side view of the screw, when the axis is parallel to the paper, as it usually is.

In regard to the groove, the case is different; and here it is to be observed that since *nj*,

the left-hand side of the next thread, is parallel to *ik*, it will cut *lm* at some point, *p*, on the vertical line through *f*, but some distance above it.

The groove, therefore, does appear sharp, though not as sharp as it really is; the bottom of it (or the actual root of the thread) is not visible; and the apparent depth, *vp*, is less than the full depth. In Fig. 98 only the upper half of the screw is shown; which however is sufficient, since if simply inverted it represents the lower half just as well.

Accordingly, in Fig. 99, which represents the entire screw, *o'k'* is parallel to *ok*, and *o'm'* parallel to *om*. In order to make the screw enter the nut readily, the thickness of the last thread is reduced by turning it off by a plane perpendicular to the axis; the end of the thread

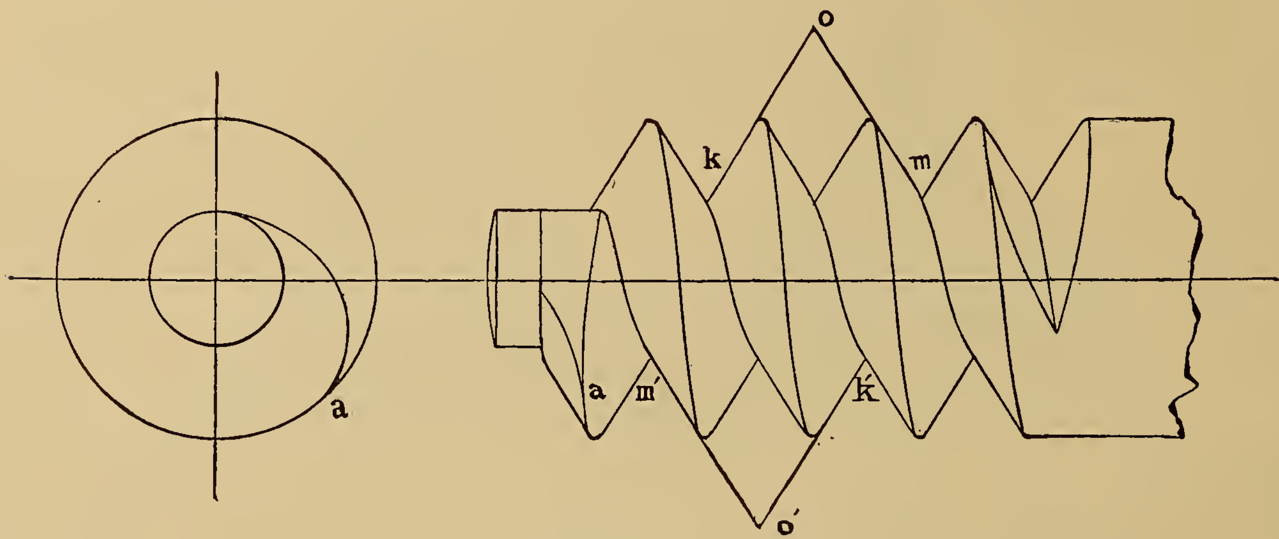


FIG. 99.

is also rounded as shown in the end view, thus producing in the side view two curves tangent to each other and to the outer helix at *a*.

162. A section of the nut is given in Fig. 100. In relation to this, it is a common error to suppose that its contour should correspond with the visible contour of the screw, as *alpng* in Fig. 98. That this is not the case will be clearly seen by imagining the nut, as before suggested, to be made by casting type-metal around the screw, which, expanding as it solidifies, must exactly fill the groove and fit the screw. A section of both screw and nut by a plane through the axis will then show but one contour-line common to the two; and this will be composed of right lines, intersecting each other, in the case here illustrated, at angles of 60°, with no rounding off or curvature of any kind whatever, as shown in the figure.

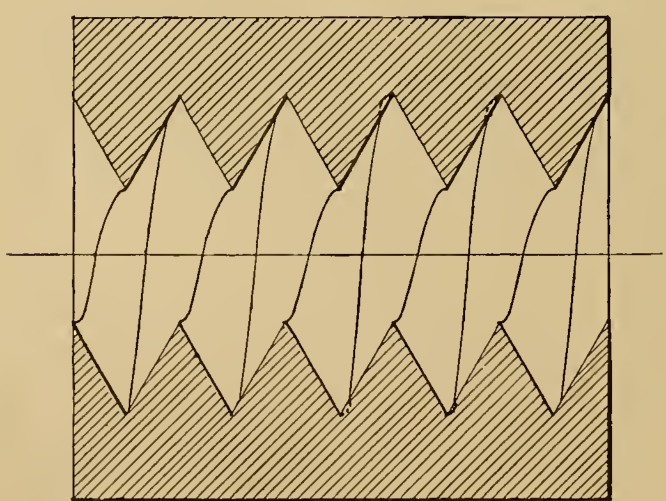


FIG. 100.

On removing the remaining half of the screw the outer and inner helices on the interior of the nut will be exposed; small portions of the

former, near the vertices, will be hidden on account of the twist of the surface, but the latter will be visible from vertex to vertex.

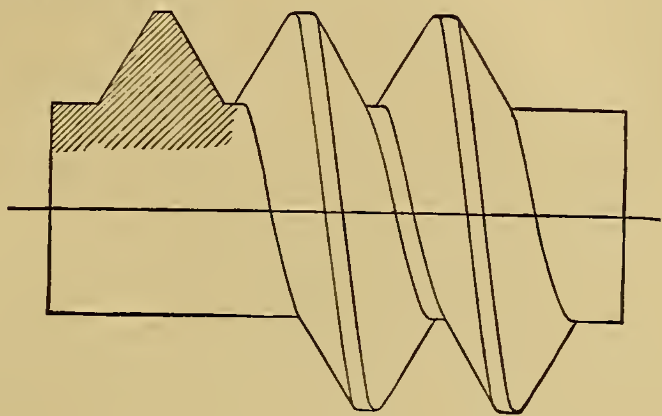


FIG. 101.

163. The threads of bolts are seldom made "full sharp," but are usually made flat on the top, the groove also being flat on the bottom, as shown in Fig. 101. In the complete drawing, therefore, the bottom of the groove is usually seen, as in the figure: the construction does not involve any considerations beyond those already presented, but it may be as well to call attention to the fact that the contour-line *ik* in Fig. 98 intersects the upper outline of the core at a point *s*, at a small but yet a perceptible distance to the left of the vertex, *e*, of the inner helix.

It need hardly be added, that V-threaded as well as square-threaded screws may be made, and sometimes are required to be, left-handed as well as right-handed, and with two, three, or any other number of threads. Screws are also sometimes made with a different slope on one side of the thread from that on the other, or even with threads whose section is bounded by curved lines. It is not, however, necessary to illustrate these variations, as due attention to the foregoing will enable the student to draw them correctly.

164. In the preceding illustrations of the application of the helix in the drawing of screws and nuts no attention has been paid to the customary proportions of diameter to pitch and depth of thread. The usual conditions have been purposely exaggerated in order to emphasize the peculiarities of the curve. And this has been done with a set purpose: in the ordinary course of mechanical drawing for constructive purposes, bolts and nuts are very seldom met with which need to be drawn with such minute precision as has been heretofore insisted on; and in Part II a chapter is devoted to the illustration of what may be called conventional methods of representing them when used, as they mostly are, merely as a means of fastening various parts together.

But the helix is encountered by the mechanical engineer on much larger scales in the construction of screw-propellers; and in laying out the drawings for these the utmost exactness in regard to this fundamental curve is necessary.

No illustration of this application is here introduced, because the drawing of even the simplest form of the propeller in use involves problems relating to the intersection of surfaces which could not be reasonably discussed at this stage. In the mean time, a thorough understanding of the processes relating to the correct delineation of the helix can be acquired, independently of the application of the curve to that or any other purpose.

CHAPTER VI.

INTERSECTIONS AND DEVELOPMENTS OF SURFACES.

165. In Fig. 102 a vertical hexagonal prism is represented as cut by a plane *pp*. Since this plane is perpendicular to the paper, the points in which it cuts the edges 1, 2, etc., are

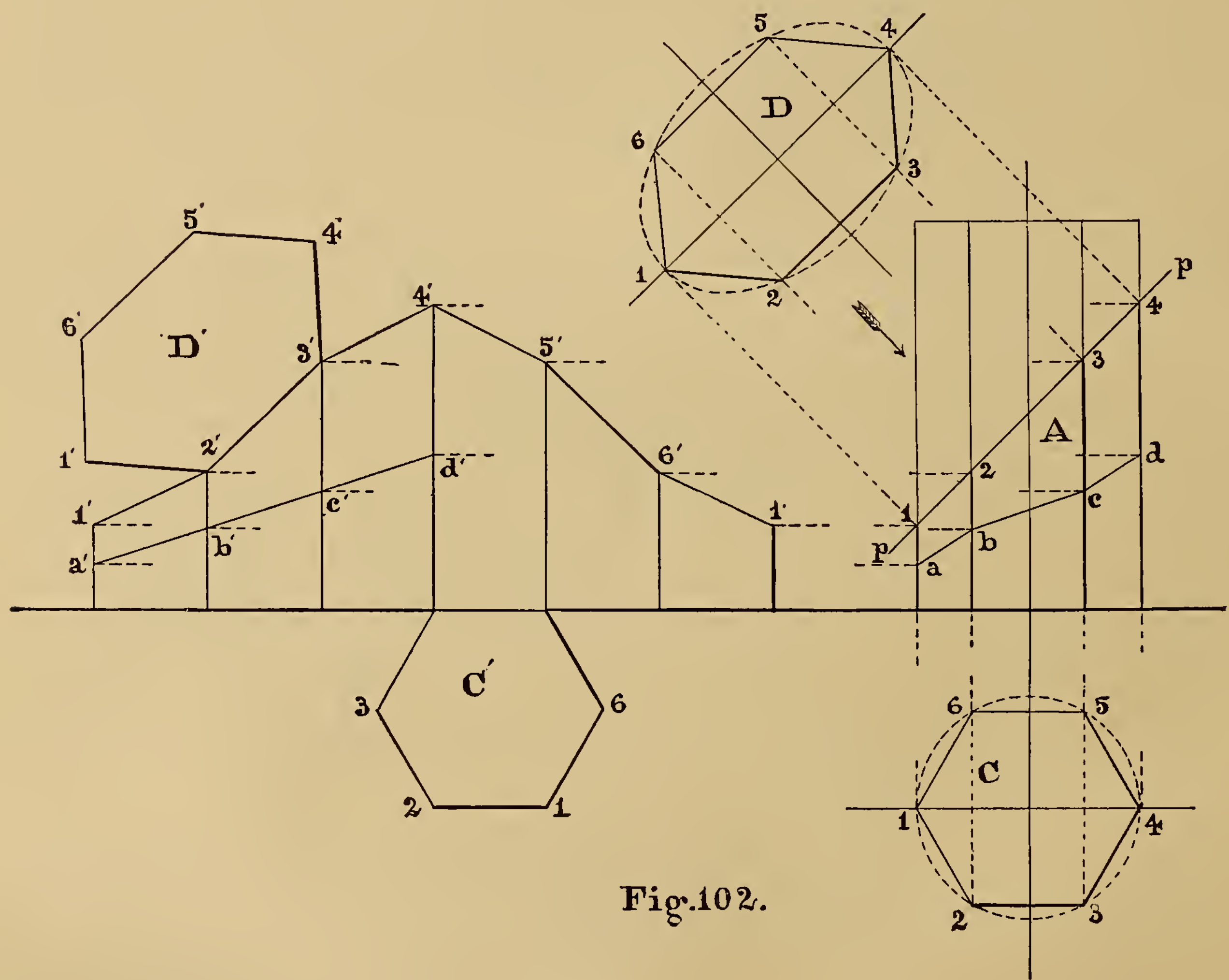


Fig. 102.

seen directly in the front view *A*, and it is self-evident that the top view *C* is not affected in any way.

The true form of the section will be seen by constructing a view, D, looking perpendicularly against the plane pp, as shown by the arrow. In this view the line 1–4 will appear of its true length and parallel to pp; and the lines 2–6, 3–5 will be perpendicular to 1–4, and also appear of their true length, which is seen in the top view C. Having located the points thus indicated, the contour of the section will be an irregular hexagon.

166. Now suppose the upper part of the prism to be removed, and the lower part to be formed of thin sheet-metal. Let this be cut vertically along the edge 1, and unfolded into a plane. Since the edges of the lower base lie and remain in the horizontal plane, they will extend themselves into one right line, whose length is equal to the perimeter of the base C, that is, to six times one of the sides. Thus, in the development shown at the left, the edges will be represented by equidistant vertical lines, and the lengths of these lines will be the same as in the front view A of the truncated prism. The upper extremities of these verticals are joined by right lines, which completes the development of the vertical surface of the prism. If desired, D', a copy of D, and C', a copy of C, may be attached as shown: the entire outline thus formed being cut out of card-board, and the lines of junction being cut half through the thickness with a sharp knife, the whole may be folded up into a very satisfactory model.

167. Let it be required to find the shortest path on the vertical surface of the prism, from the point a to the point d. In the development these points fall at a' and d'; the least distance between them is the right line $a'd'$, which cuts the edges $2'$ and $3'$ at b' and c'; project these back to the original positions of those edges in view A, and the broken line $abcd$ represents the path required.

This problem of "the shortest path" may be similarly treated in the case of any solid whose surface is capable of development into a plane, provided that in the development a straight line can be drawn between the two points.

168. If we imagine the prism in Fig. 102 to be inscribed in a circular cylinder, the circle which circumscribes the hexagon in the top view C will be the base of that cylinder, the edges of the prism will be vertical elements, and the true form of the section will be the ellipse shown in a dotted line in view D; also, the development of this section will be a curve passing through the points $1'$, $2'$, $3'$, etc., of the development of the prism.

In order to define these curves more accurately, it is necessary only to divide the circumference in the top view into a greater number of parts, and by thus increasing the number of elements, as shown in Fig. 103, to locate more points in the curves. The details of the process have already been explained in connection with Fig. 90, and need not be repeated. Also, it was shown (**149**) that the "shortest path" between two points on the surface of the cylinder is in general a helix. That is to say, a helix drawn upon a cylinder develops into a right line when the cylinder is unrolled into a plane.

169. Conversely, if an oblique right line is projected upon a cylinder, its development on unrolling the cylinder will be the projection of a helix. Thus in Fig. 103, ab represents such a line projected upon the cylinder in the front view A; draw in the development the horizontal line $a'a''$, and through b' a line ll parallel and equal to it: then the curve $a'b'a''$ is the projection of a helix whose pitch is $a'a''$, lying upon a cylinder whose diameter is $a'l$, the axis

being the horizontal xx passing through 3 in view A. Moreover, the tangent tt at $3''$ is parallel to ab or pp. The precautions and expedients mentioned in relation to the drawing of the helix therefore apply as well to the drawing of this development of the section of a cylinder by a plane.

Since the ellipse is perfectly symmetrical about both axes, it may be turned end for end; and the two parts of a cylinder thus cut by a plane making an angle of 45° with the axis may be joined together as at *V*, making what is known as a "square elbow." By using other

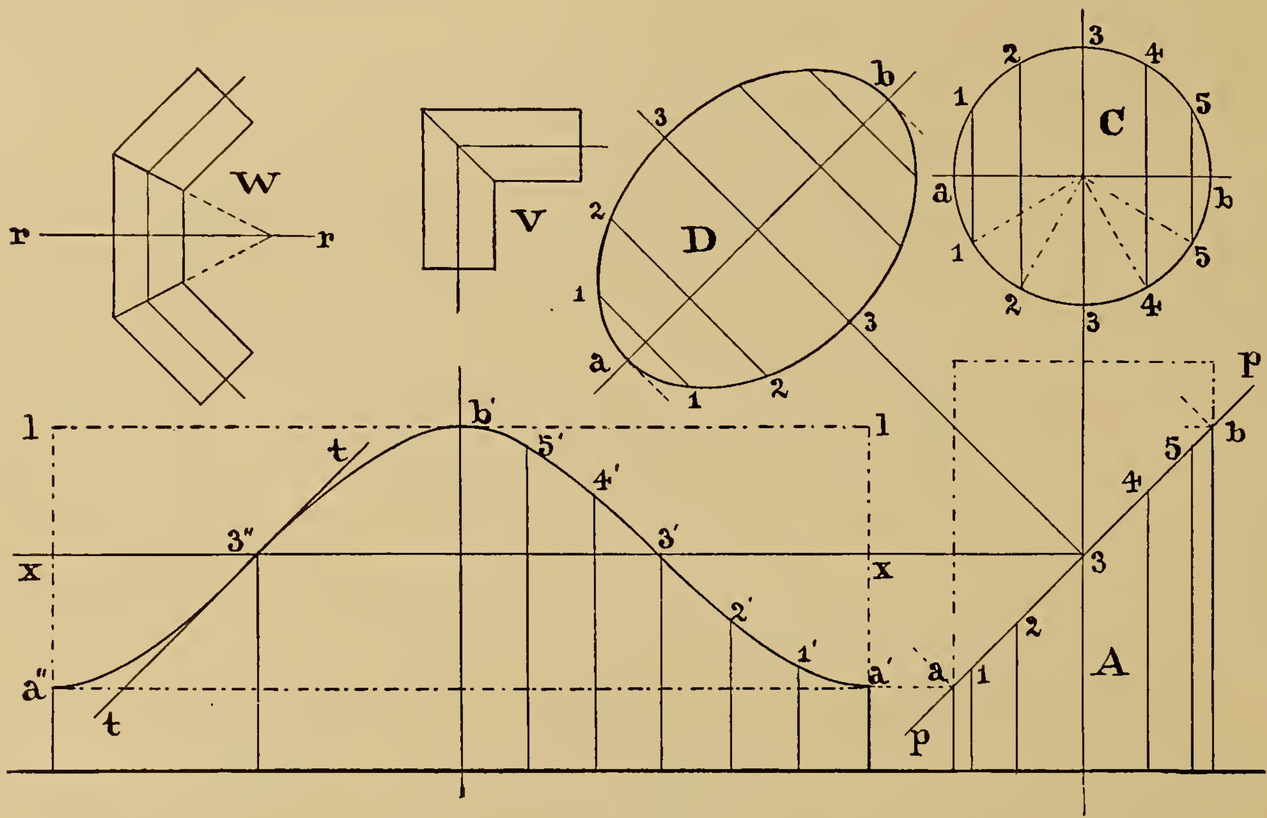


FIG. 103.

angles, the pieces may be put together at different inclinations, as shown at *W*, which represents a "three-section elbow." To lay out the sheet for the middle piece, cut it transversely by a plane rr: the circle will develop into a right line, and the ordinates are measured each way from this in order to determine the contour.

170. In Fig. 104 the hexagonal pyramid is cut by the plane pp. The top view of the section is an irregular hexagon, determined by projecting the points 1, 2, 3, etc., of the front view A vertically downward to the corresponding edges in view C; after which the view D is constructed exactly as in the case of the prism, Fig. 102.

In the development the vertex of course remains a point; and since the edges of the pyramid are of equal length, their lower extremities will lie in the circumference of a circle whose centre is v', and radius $v'n'$, equal to vn: the chords $n'm'$, $m'r'$, and so on, are equal to each other and of the true length, nm, mr, of the edges of the base.

The true distances of the points 1 and 4 from the vertex are seen in view *A*; therefore in the development $v'1' = v1$, and $v'4' = v4$. But while vm is actually equal to vn, it is foreshortened in view *A*. All parts of it are, however, foreshortened in the same proportion; therefore drawing through 2 a horizontal line cutting vn, we have $v2''$ as the true distance from 2 to the vertex, and the true length of $v3$ is found in like manner. Therefore set off $v'2' = v2''$, $v'3' = v3''$, and the broken line $1'2'3'4'$ is the development of the section of the pyramid.

The representation of the shortest path from a to d is found by reversing this process: the right line $a'd'$ cuts $v'm'$ and $v'r'$ in b' and c'; set off vb'', vc'', respectively equal to $v'b'$,

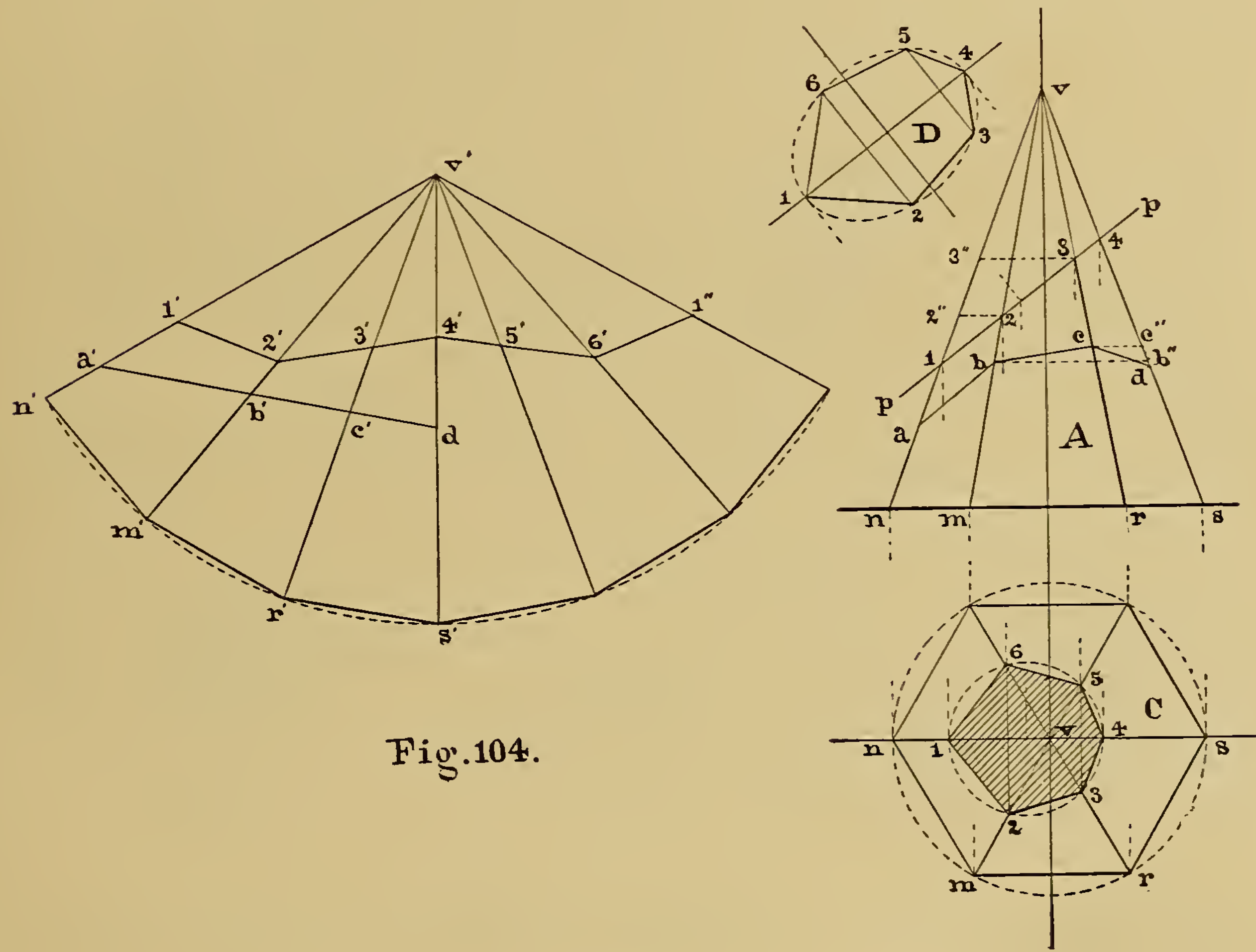


Fig. 104.

$v'c'$, on vs, and draw horizontals through b'' and c'' to locate b and c: then the broken line $abcd$ is the path required, which will also be visible in the top view.

171. If a cone be circumscribed about the pyramid, its base will be the dotted circle in the top view of Fig. 104, and the edges of the pyramid being rectilinear elements of the cone, the true form of the section will be the dotted ellipse in the view *D*: this section will also be projected as an ellipse in the view *C*, and in the development will form a curve passing through the points 1′, 2′, etc.

The construction in the case of the cone is shown in Fig. 105: if we draw any rectilinear elements, as *vm*, *vr*, they will be cut by the plane at points 1, 2, seen directly in the front view, whence they may be projected to the top view. But in this top view these projecting lines will cut the elements very acutely in the neighborhood of *vs*, so that the curve cannot thus be very accurately defined. In this region another process is preferable: suppose the cone cut across transversely by a plane *lf*; the section will be a circle, which may be drawn in the top view. Since the planes *lf*, *pp* are both perpendicular to the paper in the front view, the point *q* represents their intersection, which in the top view will be seen in its true

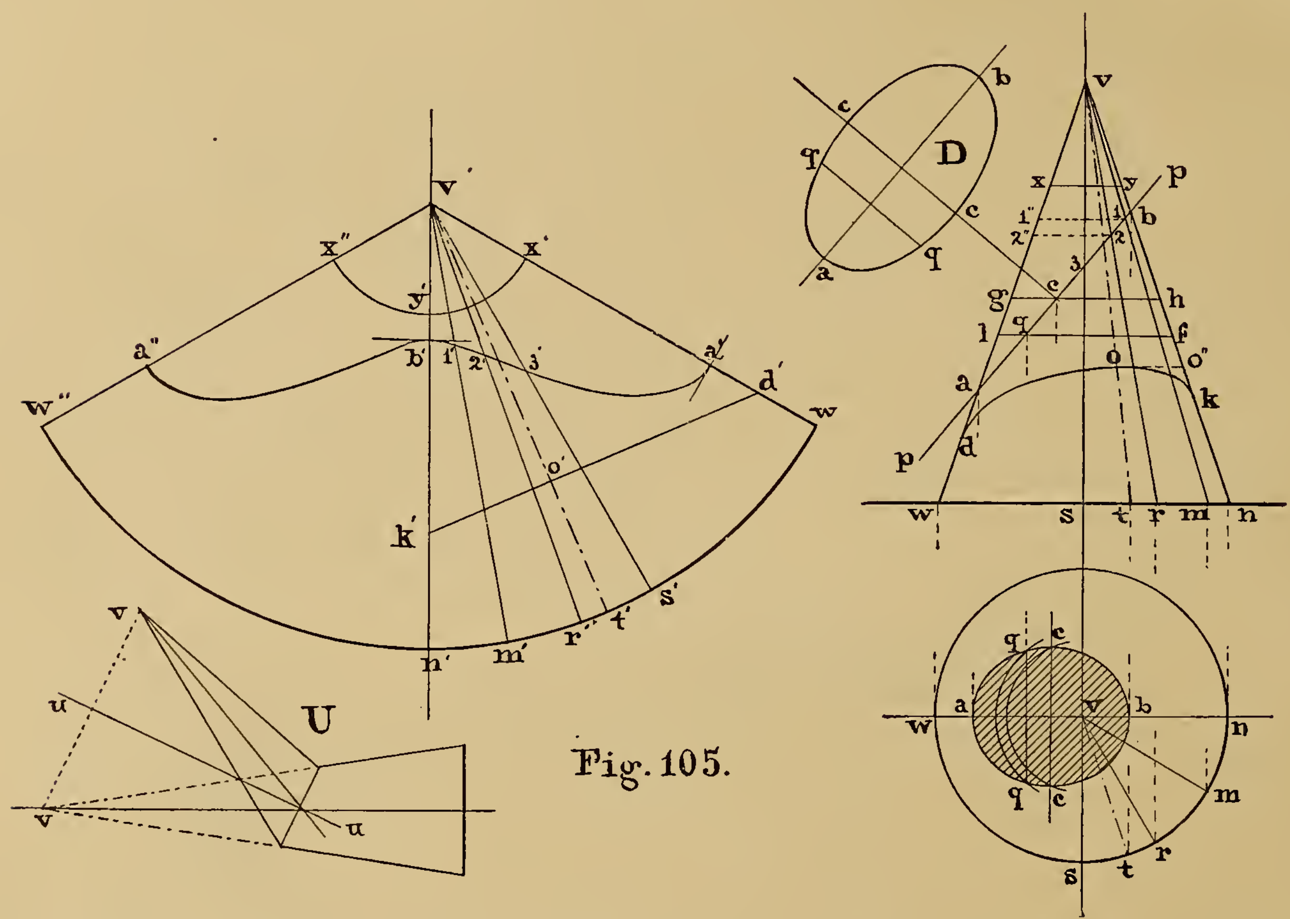


Fig. 105.

length as a chord *qq* of the circle just drawn. This operation may be repeated as many times as is necessary: and it is needless to say that any such chord will also be seen in its true length in the view *D*.

172. Since the section of the cone by the plane is known to be an ellipse (when, as in this case, all the elements are cut), it is perfectly legitimate to determine the axes, and construct the curve by any convenient method. It is clear that *ab* in the front view is the length of the major axis in the view *D*; to determine the minor axis, bisect *ab*, and pass a horizontal plane *gh* through the point of bisection, by means of which find the chord *cc* in the top view as just explained: this will be the minor axis of the ellipse seen in the top view, as well as of the true elliptical section *D*.

It is clear that, as in the case of the cylinder, the ellipse may be turned end for end, and the upper part of the cone joined to the lower, as in the small diagram U; the vertex going from its original position v to the new one v', by revolving around an axis uu, perpendicular to the plane of the ellipse at its centre.

173. The developed surface of the entire cone will be limited by two rectilinear elements, including a circular arc whose radius is the slant height, and of a length equal to the circumference of the base. Having described with that radius an indefinite arc about v', lay off upon it, by the process of Fig. 34, an arc equal in length to a convenient fraction of the circumference of the base, rectified as in Fig. 33, and step this off the required number of times, to determine $w'n'w''$: this done, points in the curve $a'b'a''$ are located precisely as in the case of the pyramid, Fig. 104. In relation to this, it is to be particularly noted that as the tangents to the ellipse at a and b are perpendicular to the elements through those points, so the tangents to the developed curve are perpendicular to $v'a'$, $v'a''$, $v'b'$.

The "shortest path" from d to k is determined exactly as in the case of the pyramid. Since in the development $d'k'$ cuts both $v'w'$ and $v'n'$ obliquely, it is to be observed that in the front view the projection of this path will be tangent to vw at d, and to vn at k. The highest point of this projection may be found as follows: The least distance of $k'd'$ from the vertex is the perpendicular $v'o'$ let fall from v' in the development; set off $vo'' = v'o'$, on vn in the front view, and a horizontal through o'' will be tangent to the path at its highest point. Produce $v'o'$ to t', rectify the arc $n't'$, and set off an arc nt of the same length in the top view; project t to the line of the base in the front view: then tv will be the element corresponding to $t'v'$ in the development, and will cut the horizontal through o'' in o, the required point of tangency.

174. These problems of development are of direct practical use to the workers in sheet-metal. Thus if the cone be cut off by another horizontal plane xy, the frustum $wxyn$ forms the body of a funnel; if in Fig. 103 the plane pp cuts the elements of the cylinder at an angle of 45°, the two portions of the solid may be joined to form the "square elbow" V; and the development is the shape of the metal sheet from which each piece is made. By using other angles, the cylindrical pieces may be joined as at W in Fig. 103; in this case an imaginary right section at rr will develop into a right line, and by measuring each way from this the contour is determined.

Theoretically, it makes no difference along what element the cylinder or cone is cut; but practically it should, when there is no reason to the contrary, be cut *so as to form the shortest seam.*

Again, the drawing of a bevel-wheel involves the development of a cone, upon which the outlines of teeth are laid out; the correct representation of the teeth of the wheel depending upon the forms which these outlines assume when the developed sheet is re-formed into its original cone.

175. It will be understood from the preceding examples that in the development of a surface by unfolding or unrolling it into a plane there is no extension or compression, or distortion of any kind. Lines of course change their forms, but not their lengths: if they intersect on the original surface, they will after development intersect at the same angle; if, on

the other hand, they are originally tangent, they will remain tangent. The surfaces of solids bounded by planes are developed by unfolding, the various faces turning upon the edges as upon hinges; the surfaces of cylinders and cones, as we have seen, are developed by unrolling, the only difference being that the number of edges is infinite. These are the only curved surfaces commonly met with that are capable of development; but there is a third kind, of which a single example will be sufficient.

176. It is a familiar fact that if a piece of paper in the form of a right-angled triangle, as $AA'B'$, Fig. 90, be wound around a cylinder so that the base of the triangle forms the circumference of the base of the cylinder, the hypothenuse will form a helix: thus AB' of Fig. 90 becomes the helix $A369B$ of Fig. 89. In Fig. 106 the paper is shown as partly unwound; bm being equal to the quadrant $a'c'$ of the base, it is apparent that the hypothenuse bc is tangent to the helix at c, and equal in length to the helical arc ac. It is clear that as the unwinding progresses the point b will trace in the plane of the base the involute $a'b'r'$ of the circumference; the upper edge of the paper will in every position be tangent to the helix, and these different tangents must lie upon a continuous surface, of which a limited portion is represented in Fig. 107. It will be recognized that this surface somewhat resembles that of a cone; but as the elements, instead of meeting in one point, are tangent at various points to the helix, the surface goes on, expanding as it rises, and winding about the cylinder in convolutions like those of a sea-shell.

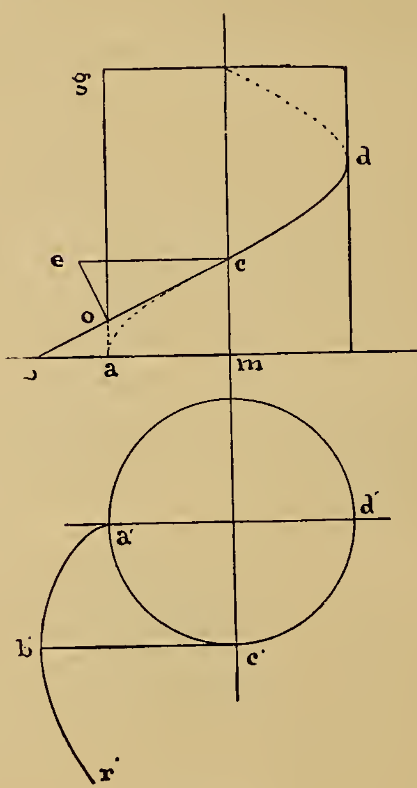


FIG. 106.

177. The helix itself, winding equably around the cylinder, has everywhere the same curvature. In Fig. 106 bc cuts ag, the projected outline of the cylinder, at o; draw at this point a perpendicular to bc, cutting at e a horizontal line through c: then ce is the radius of curvature of the helix acd.

Ascertain in this way the radius of curvature, cy, of the helix in Fig. 107; and about any point O, Fig. 108, describe a circle with this radius. If then the helix be supposed to be made even of wire having a sensible thickness, a portion of it equal in length to the circumference of this circle can "have the twist taken out of it," and be made into a ring which will lie flat on the paper; and all the tangents to the helix will also lie in the same plane, and be tangent to the circle. Since the length of each tangent, in the formation of the surface, was equal to that of the helical arc from which it was unwound, and remains the same after the development, the involute $a'b'f$ in Fig. 107 will become the involute of the new circle in Fig. 108. Making the tangent AH in this figure equal in length to the circumference, then, the piece $AHGFBA$ may be cut out of paper or thin metal, and (the circle being also cut out) it can be bent up, the circumference fitting the helix $acdc$ in Fig. 107, the curve $ABFG$ at the same time contracting into the involute $a'b'f'g'$, and thus the plane will assume the form of the

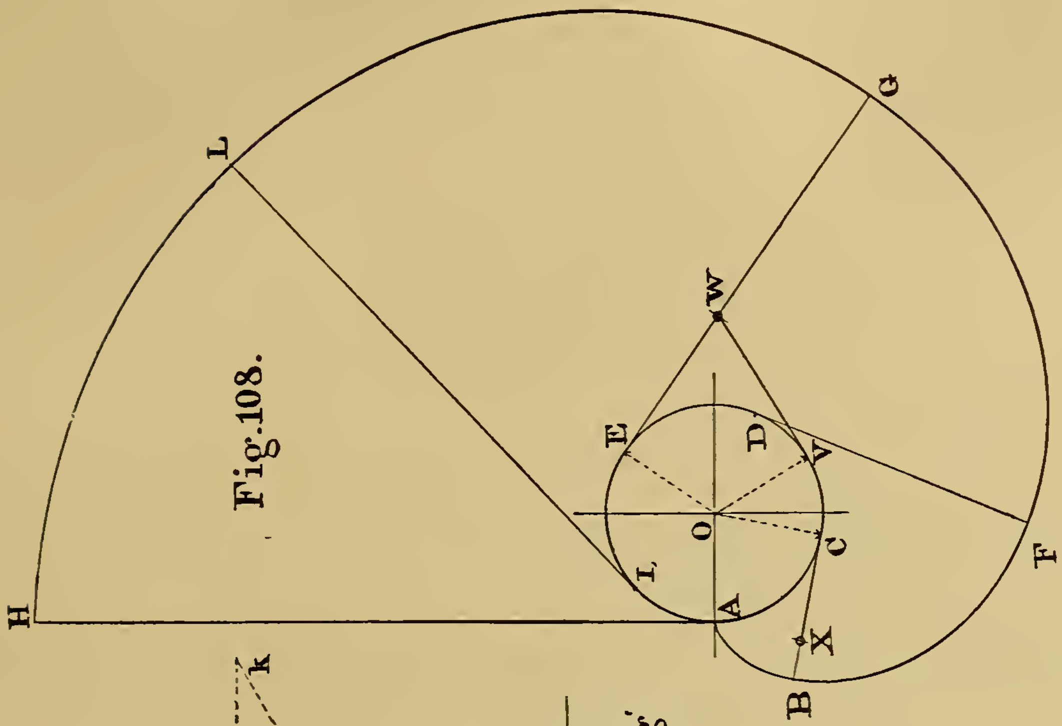


Fig. 108.

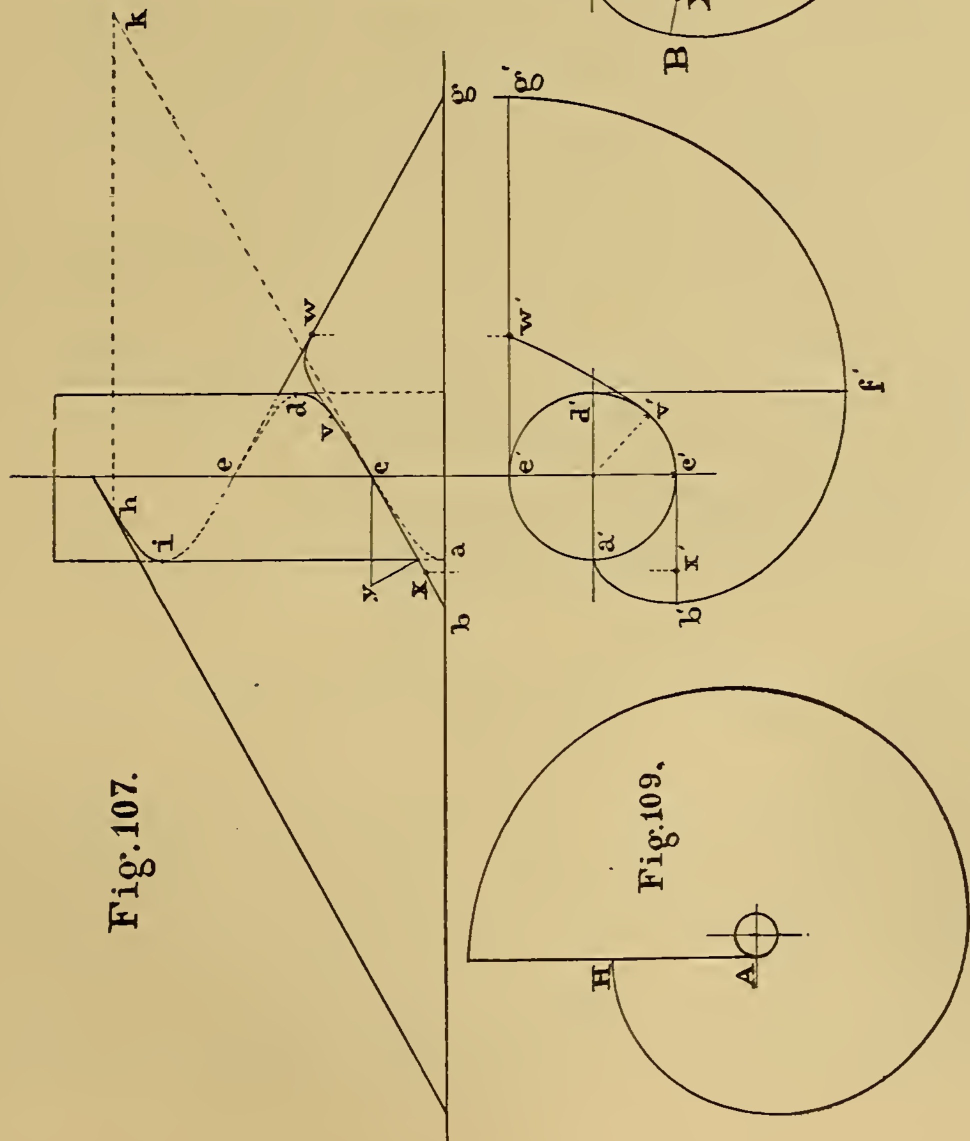


Fig. 107.

Fig. 109.

original surface. Produce bc in Fig. 107, set off $bk = AH$, and draw a horizontal through k, cutting the helix in h, which will be the limit to the height of the part of the surface of which Fig. 108 is the development. No more than this can be cut out of a single sheet; but since the edge HA is lifted out of the plane in forming up the surface, another piece, shown on a smaller scale in Fig. 109, can be joined to the first one along that edge, and so on indefinitely.

178. Let it be required to find the shortest path upon the surface from x to w, in Fig. 107. Since x lies upon bc, its position X in Fig. 108 will be upon the corresponding tangent; make AC equal to the helical arc ac: then the tangent CB is the one required. Similarly, make $ACDE$ equal to the helical arc $acde$: then the tangent EG is the position of eg in the development, and upon it W is located.

We now find that a straight line cannot be drawn upon this development from X to W, because such a line would cross the circle, within which the surface has no existence, thus presenting a singular exception to the general rule. Draw WV tangent to the circumference: then the shortest path is composed of the two tangents XC, VW, and the circular arc CV between the two points of tangency.

On the original surface, then, this path is made up of the portion xc of the element passing through x; of the portion cv of the helix; and of a curve vw tangent to the helix,—which can be constructed by drawing in Fig. 108 tangents to the arc VE, as DF for instance: these are the developed positions of elements, and cut VW in points which, when they are restored to their original locations on the surface, lie upon the required line.

179. From the preceding examples it will have been gathered that in order to construct the intersection of a surface with a plane it is necessary to find the points in which the plane cuts a series of lines drawn upon the surface: and it is clear that these lines should be the simplest that the nature of the case will admit of, unless the intersections are too acute to give reliable determinations; which may happen, as we have seen in the case of the cone, when the rectilinear elements alone are depended on. Also, it will be noted that the object and the plane are represented in such a position that in one of the views the plane is seen edgewise: this greatly facilitates the construction, and exhibits most clearly the reason for each step. If after the section is made a view of the object from any other direction or in any other position is needed for any reason, it can readily be made by the processes explained in Chap. IV.

180. Fig. 110 represents a solid of revolution; that is to say, one that can be turned in the lathe; every transverse section is a circle, which in general is the simplest line that can be drawn on such a solid.

In determining the intersection of this object with the plane pp these circles are made use of precisely as in the case of the cone (**171**). Thus the point a in the front view represents a chord in the circle whose radius is ik, and this chord is seen in its true length in the top view and also in the view D, which shows the exact outline of the section. So, again, c and d in the front view represent the chords cc, dd, in the circle whose radius is lm: e in the front view coincides with d, but represents a chord ee of the circle whose radius is ln, n being the point at which the curved outline of the base of the object is tangent to the prolongation of the

horizontal line *lm*. Thus, since the upper side of the base has a narrow *plane* ring whose breadth is *mn*, the small portions *de*, *de* in the section are straight lines.

It is hardly necessary to point out that in actually constructing the section very small arcs only of the circles used need be drawn, and the chords should not be drawn at all, the extremities alone being marked.

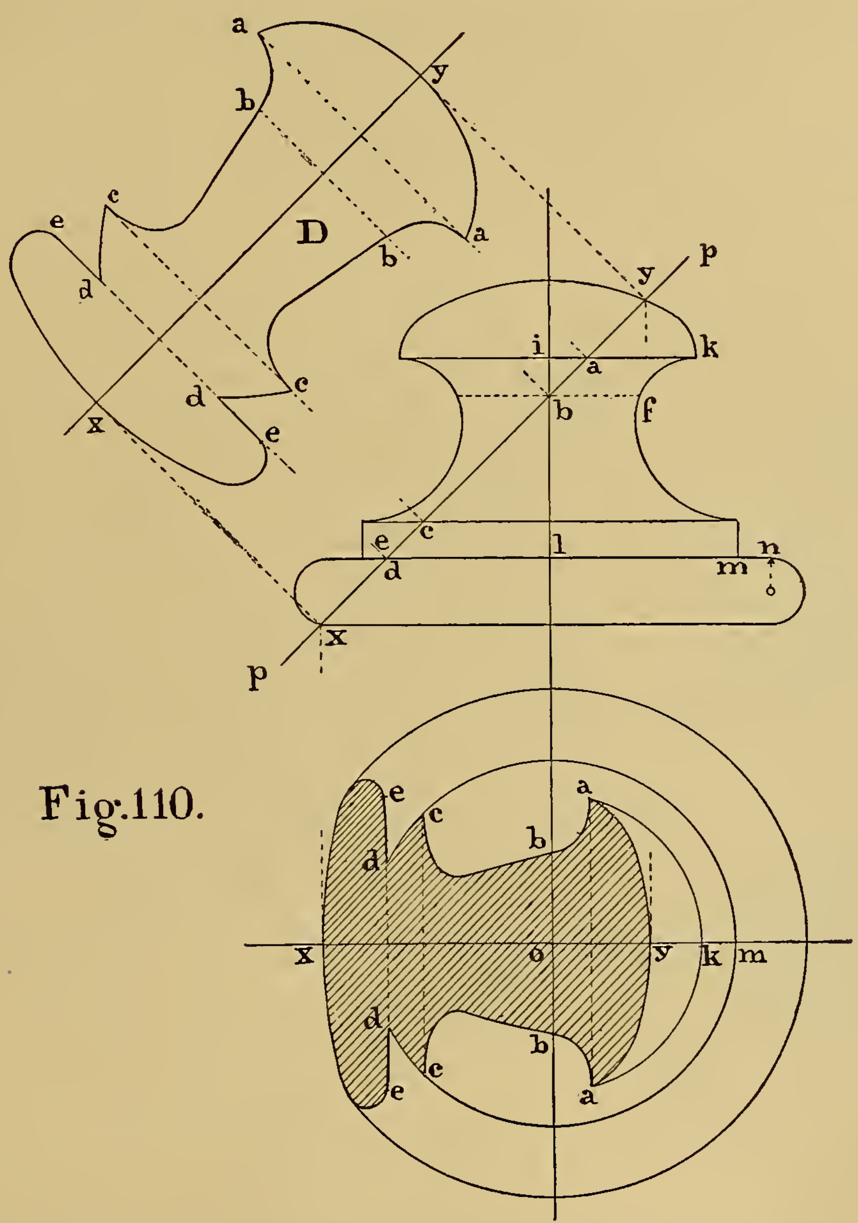


Fig. 110.

181. Fig. 111 represents also a solid of revolution, whose outline *gif* is an hyperbola whose conjugate axis *ll* is the axis of the solid. It may also be formed by the revolution about the same axis of the two lines *aob*, *cod*, which in the front view coincide with the asymptotes: these are in this view parallel to the paper, and in the top view coincide in one line *a′b′*, the distance *l′o′* being equal to *io*, the semi-major axis; in the side view *B* they also coincide in one line *bc*, and obviously they determine a plane parallel to *ll*, which, while cutting these two lines out of the surface, is also tangent to it at the point *o*.

It is apparent that the point b describes the circle of the lower base, whose radius is $l'b'$; also that o describes the *circle of the gorge*, ii, its radius $l'o'$ being also taken directly from the top view. Every intermediate point on ab follows the same law, so that if this line and the axis are given or assumed, the hyperbolic outline is most expeditiously drawn by the method of Fig. 48.

The section of this surface by an inclined plane pp which cuts all the elements is an ellipse, of which the major axis is xy. Bisect xy at r; a transverse section through this point

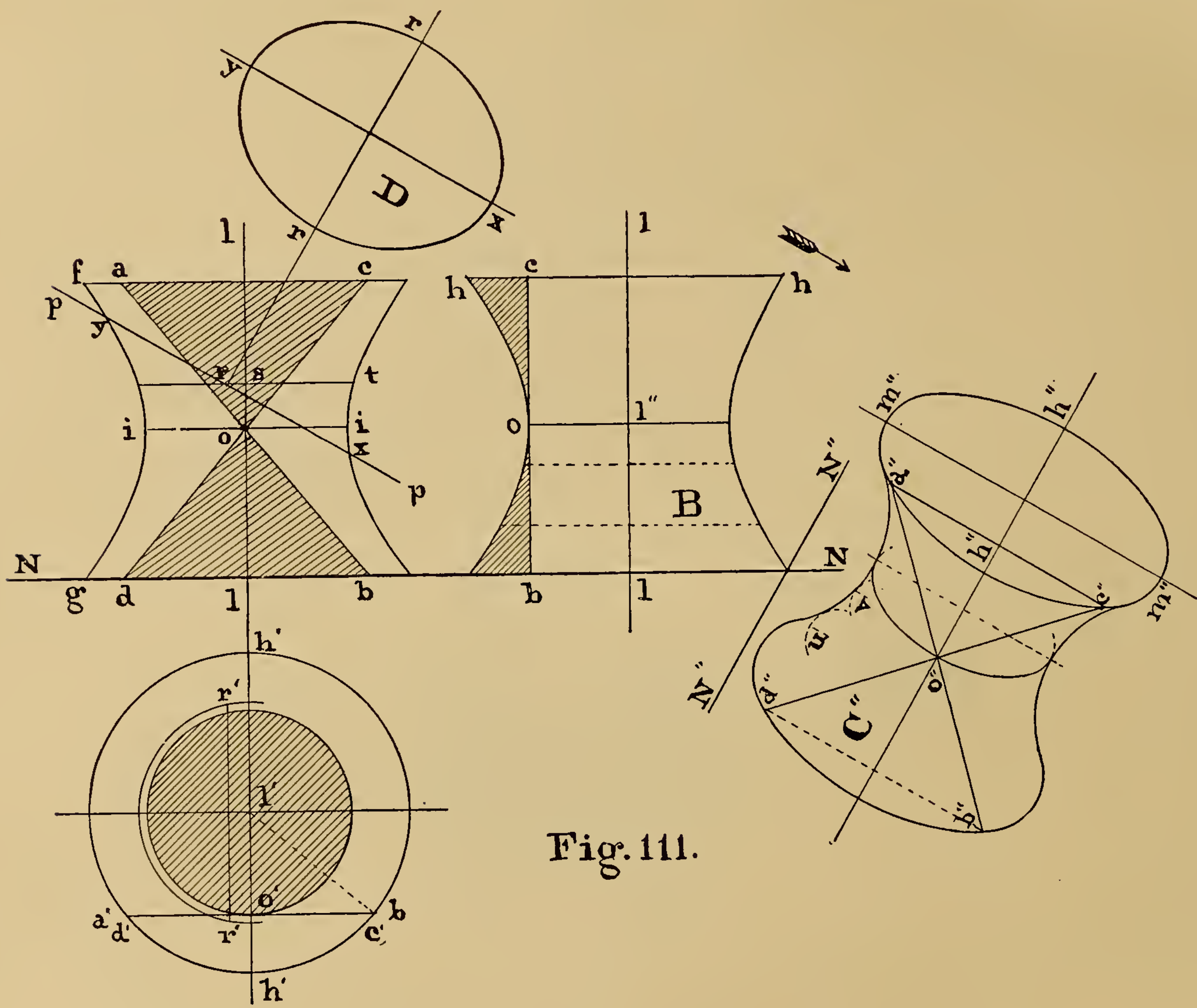


Fig. 111.

is a circle with radius st, in which r represents the horizontal chord $r'r'$ in the top view, and this is the minor axis; from which data the view D is constructed directly.

182. Drawing a new base-line $N''N''$, let it be required to make a view C'', looking perpendicularly down upon the new horizontal plane thus indicated, as shown by the arrow. The axis will appear as a line parallel to $N''N''$, and the upper base as an ellipse of which the minor axis is $h''h''$, the foreshortened projection of hh, the major axis $m''m''$ being seen in its true length, that is, equal to hh. The lower base will appear as a similar and equal ellipse, and the circle of the gorge as a smaller one, constructed in like manner. So also any inter-

mediate transverse sections will be projected as ellipses, small portions of which are shown at u, v, and the outline will be limited by a curve tangent to all these ellipses.

This illustrates very clearly an important principle of universal application, viz., that **the visible contour** of any object, in any projection whatever, is **the envelope of all the lines which can be drawn upon its surface.**

183. Fig. 112 represents the "stub end" of a connecting-rod; it is rectangular in section, as shown by $f'kmn$ in the end view, and is joined to the cylindrical neck of the rod by the surface of revolution, whose outline WY is usually, as here shown, a circular arc. We have, then, to determine the intersections of this surface by two planes pp, rr, parallel to the axis.

A transverse section at a is a circle, which in the end view is seen to be cut by the plane

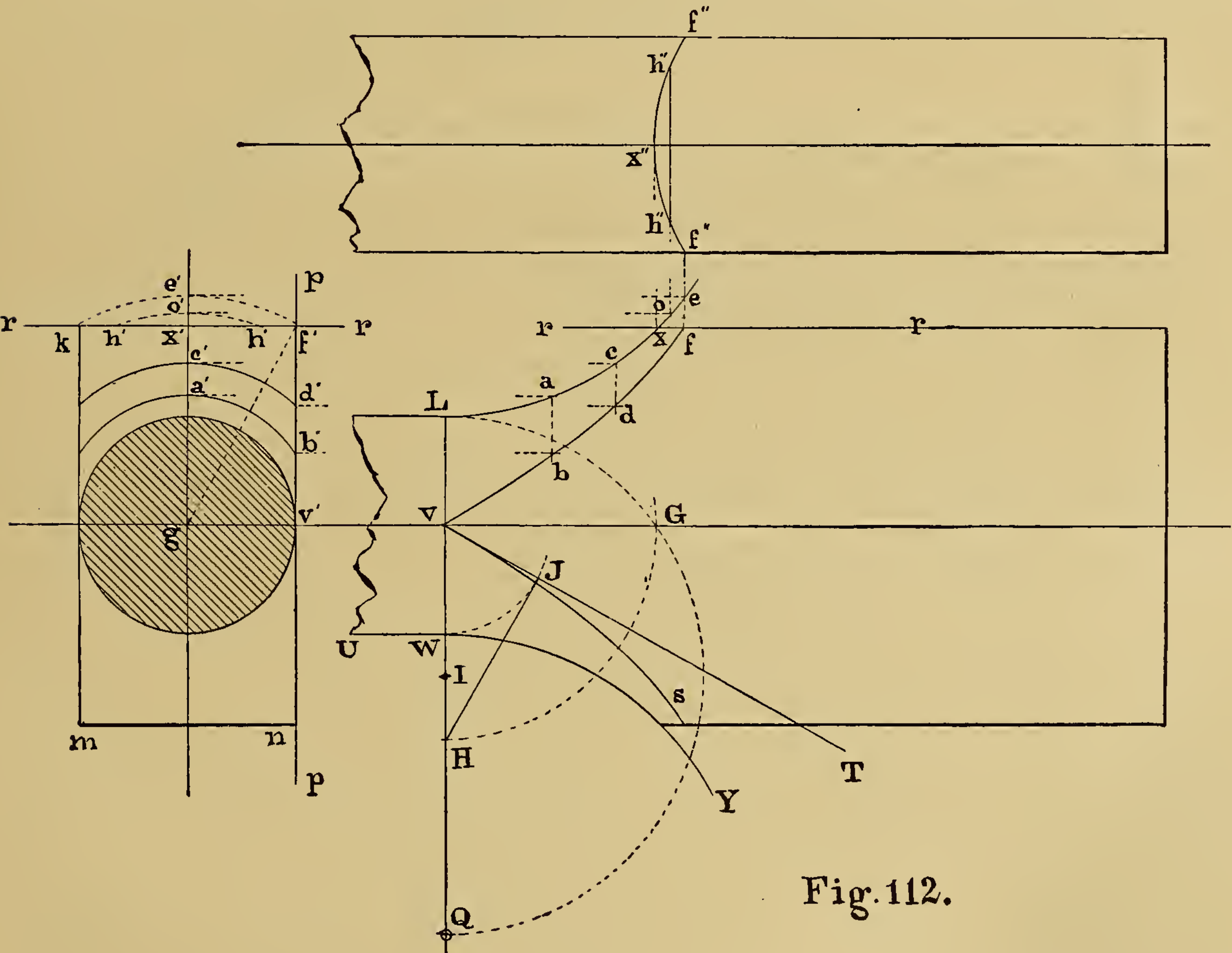


Fig. 112.

pp at b', which is projected back to b, a point in the required curve; in like manner the point d is located by means of a transverse section at c, and so on.

The plane rr cuts the outline at x, which determines the vertex x'' of the curve seen in the top view. This plane intersects pp in the line forming the upper front edge of the stub end, which appears in the end view as the point f'. A circle described about g through f' cuts the vertical centre-line in e', which, projected back to the contour at e, gives the location of a transverse section from which, by the preceding process, would be determined the point f in the front view, corresponding to f'' in the top view. A transverse section at any point as

o, between x and e, is a circle, in which $h'h'$ in the end view is a chord, seen in its true length as $h''h''$, in the top view, perpendicularly above o.

184. Fig. 112 represents the case, very common in practice, in which the thickness of the stub end is equal to the diameter of the neck of the rod, so that the plane pp is tangent to the neck, as seen in the end view, and also to the concavo-convex surface of revolution at the point v. In this case the two symmetrical curves $vbdf$, vs intersect acutely at v; the angle of intersection may be determined as follows: Let Q be the centre of the contour curve WY; produce QW to L, bisect QL at I, and upon it as diameter describe a semicircle cutting the axis in G. On vQ set off $vH = vG$; about v describe an arc with radius vW, and draw HJ tangent to it: then vJT, perpendicular to HJ, is tangent to vs at v.

If the stub end is thicker than the neck of the rod, the intersection by the side plane will be a continuous curve, as fvs in Fig. 113, which will have a vertical tangent at v, and will

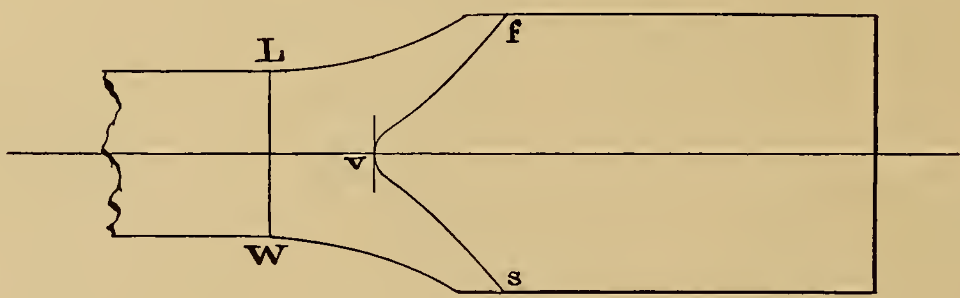


FIG. 113.

usually have a point of contrary flexure, as here shown; the case is analogous to that presented by the plane rr in Fig. 112, and requires no farther explanation. If the section of the stub end be square, as it sometimes is, the curves on all four sides will of course be exactly alike.

185. Approximate Developments.—It has already been pointed out that only surfaces of single curvature, such as the cone and cylinder, are capable of development: a warped or twisted surface like that of a screw, or a double-curved one like that of an egg, cannot be transformed into a plane without distortion. Still, approximations may be made, and are often of use, as in the laying out of maps, and in determining the forms of boards for covering hemispherical domes.

This last may be accomplished in two ways illustrated in Fig. 114. In the first method the boards are laid horizontally, forming zones, represented in side elevation and in plan in quadrants I. and II., respectively. Let AB, in the enlarged diagram on the left, be half the arc measuring one of these zones, and AC the thickness of the board. When in place, it is evident that the edge DB should coincide with a radius DL of the hemisphere, just as CA coincides with a radius CG. At C and A draw perpendiculars to CG, and on them set off CF, CE, respectively equal to the arcs CD, AB, and draw the radial line FEH: then $AEFC$ is one half of the transverse section of the board.

Let a in quadrant I. correspond to A in the large diagram: at this point draw a tangent to the outline of the hemisphere, and produce it to cut the axis in o; on ao set off ab', ae', respectively equal to AB and AE, and through the points b', a, e' describe circular arcs with

centre *o*. In the illustration it is assumed that the board is to cover one quarter of the zone: therefore the arc *al* is made equal to the quadrant *al*, *b'k'* equal to the quadrant *bk*, and the length of *e'i'* equal to that of the quadrant *ci*. This gives the approximate development for the inner surface of the board: the breadth of the outer surface, indicated by the exterior lines, is of course equal to twice *CF*.

186. In the second method the hemisphere is covered with boards cut in the form of gores, the edges, when in place, forming meridians extending from the equator to the pole, as shown in plan and elevation in quadrants III. and IV.

To lay out a gore, erect the vertical *r'p'*, equal in length to the quadrant *rp*, and divide

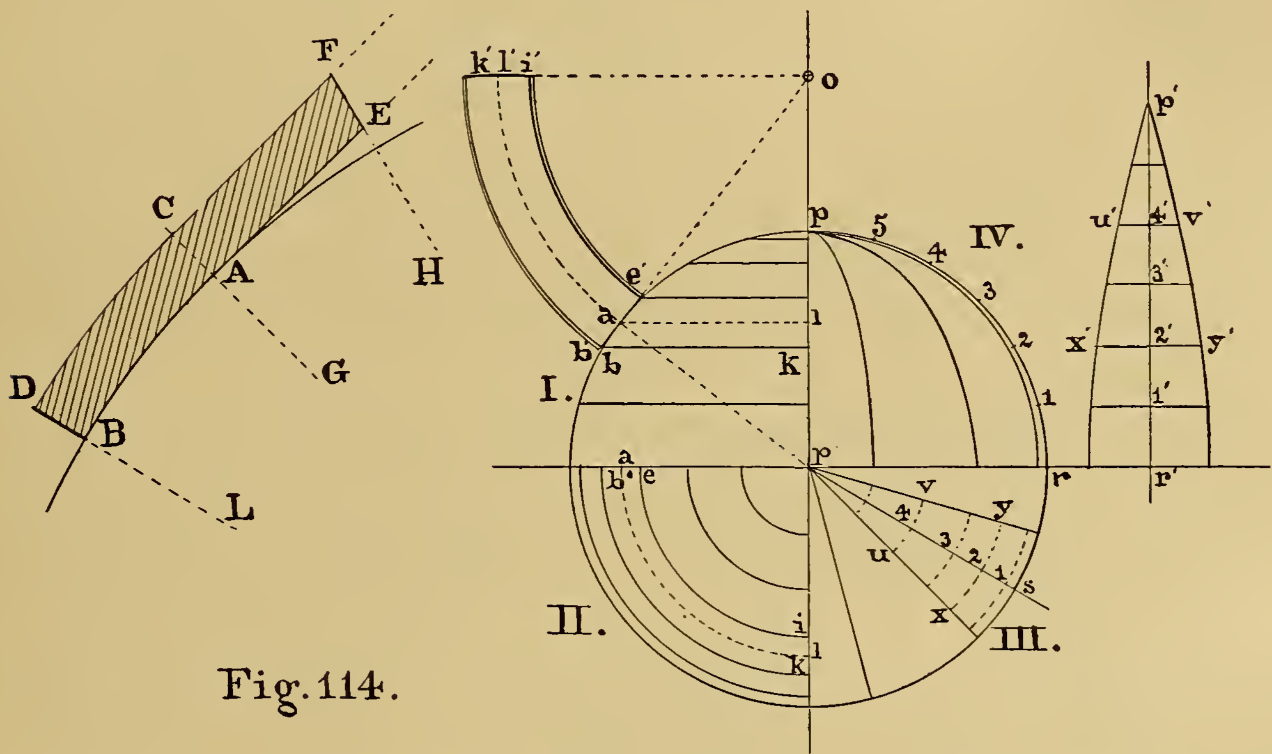


Fig. 114.

it into equal parts, corresponding to the divisions of *rp* by the points 1, 2, 3, etc. In the plan these latter points are for convenience represented upon the bisector *ps*, and through them are drawn circular arcs terminating in the edges of the gore. Then in the development make the horizontal *x'y'*, at 2', equal in length to the arc *x*2*y* in the plan; make *u'*4'*v'* equal to the arc *u*4*v*, and so on: the extremities of these horizontals are points in the contour required.

It need hardly be stated that in the application of either method the zones or the gores, as the case may be, should be narrow, and the boards thin; the breadths have been purposely made much greater than they could be in practice, in order to exhibit the processes clearly: their numbers should be at least four times as great, to give satisfactory results.

INTERSECTIONS OF SURFACES.

187. In Fig. 115 is shown a vertical cylinder, penetrated on the left by an inclined one, the two axes meeting, and on the right by a horizontal one, whose axis is not in the same plane with the other two, but a little in front of that plane.

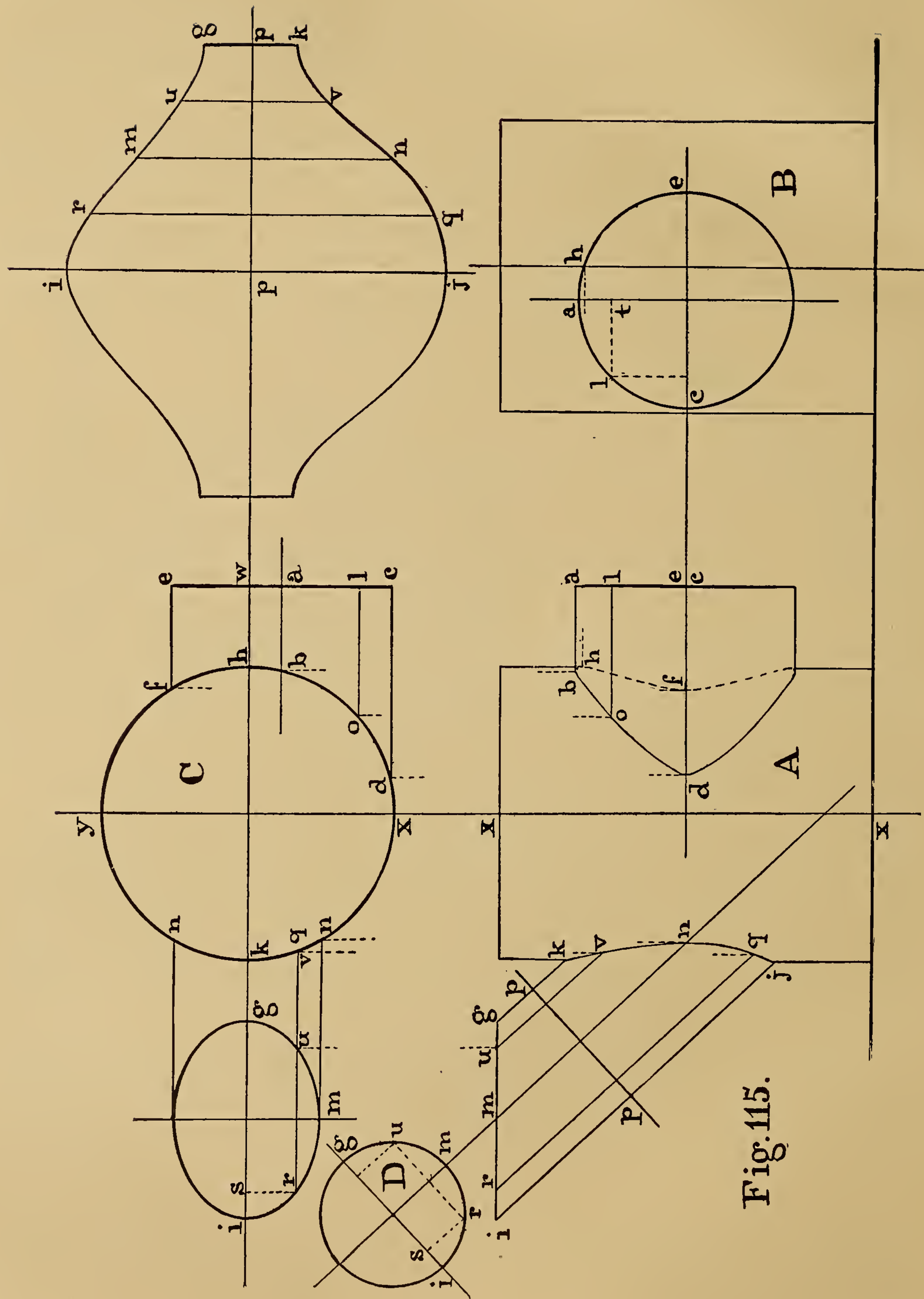


Fig. 115.

Beginning with the latter, we observe that the point *a* in view *B* represents the line *ab* in views *A* and *C*. And in the top view we perceive that this line pierces the vertical cylinder at *b*, which projected to the front view determines one point in the curve of intersection. Similarly, *c* and *e* in view *B* represent the front and rear elements, and in view *C* these elements pierce the cylinder at *d* and *f*, which projected to view *A* determine the vertices, at which the curve has vertical tangents. If in the top view we draw any element as *lo*, it will be represented in view *B* by a point *l*, whose distance *lt* from the vertical plane through *a* is equal to *la* in view *C*: this determines the height of *lo* in view *A*, to which line the point *o* is projected from view *C*, as before, and so on for as many points as may be required.

Clearly, *b* is the highest point, at which the curve is tangent to *ab*, but it is not the extreme right-hand point, which is found thus: the right-hand element of the vertical cylinder is in view *B* coincident with the centre-line, which pierces the horizontal cylinder in *h*, which projected to view *A* locates the point in question.

188. In the case of the inclined cylinder, the highest and lowest element are at once seen to intersect the left-hand element of the vertical cylinder at *k* and *j*; the nearest element is seen in view *C* to pierce it at *n*, which is projected to view *A*, as before. This inclined cylinder is circular, as shown by the direct end view *D*; being cut off at the upper end by a horizontal plane, the section appears as an ellipse in view *C*. But this does not affect the mode of operation: if we draw upon it in the top view any element as *uv*, the point *u* is projected to *ig* in view *A*, a parallel to the axis is drawn through that projection, to which the point *v* is projected from *C*, thus locating a point in the curve.

Producing *vu* in the top view to cut the ellipse in *r*, it will be seen that there is another element directly beneath *vu*, on the lower part of the cylinder, from which the point *q* is determined in the same way.

189. The development of the inclined cylinder is also shown in Fig. 115. A transverse section at *pp* will be a circle, and will develope into a right line, from which the distances to the points *i*, *j*, *g*, *k*, *r*, *q*, etc., will be the same as their distances from *pp* in the front view. In the development *pp* is one half the circumference in view *D*; and to save labor it is advisable to subdivide the semi-circumference into equal parts, and draw in *A* elements corresponding to these points of subdivision; then *pp* in the development is to be divided in the same manner: thus in the illustration the semicircle *img*, view *D*, and its rectification *pp* in the development are each divided into four equal parts. This cylinder being cut along the upper element *gk*, not only is the seam the shortest possible, but the development is bounded by curves principally convex; which is a great practical advantage in cutting out a metal sheet: it is also symmetrical with respect to the vertical *ij*, which greatly facilitates the drawing of the outline.

190. In the development of the horizontal cylinder, shown in the upper part of Fig. 116, symmetry is secured by cutting the pipe along the rear element *cf* in view *C* of the preceding figure: it is not the very shortest, but the difference is not great enough to cause any practical difficulty. In this development *cc*, of course, is equal in length to the semi-circumference *cae* in view *B* of Fig. 115; *a* is the middle point of *ce*; and *cd*, *ab*, *wh*, *ef*, the ordinates of the curve, are equal to the corresponding elements in view *C*, the distance *aw* being equal to the arc *ah* in view *B*.

These ordinates, it is to be understood, are given only for illustration ; in practice the proper course is to divide the semicircle *cae*, and its rectification *ce* in the development, into the same number of equal parts, draw corresponding elements in view *C*, and set up the corresponding ordinates of the lengths thus determined, as above advised in reference to the inclined cylinder.

191. In developing the vertical cylinder of Fig. 115, the "shortest seam" would be made by cutting it down along either the right-hand or the left-hand element ; but this would lead to the very awkward result that a part of one or the other of the openings would be in one

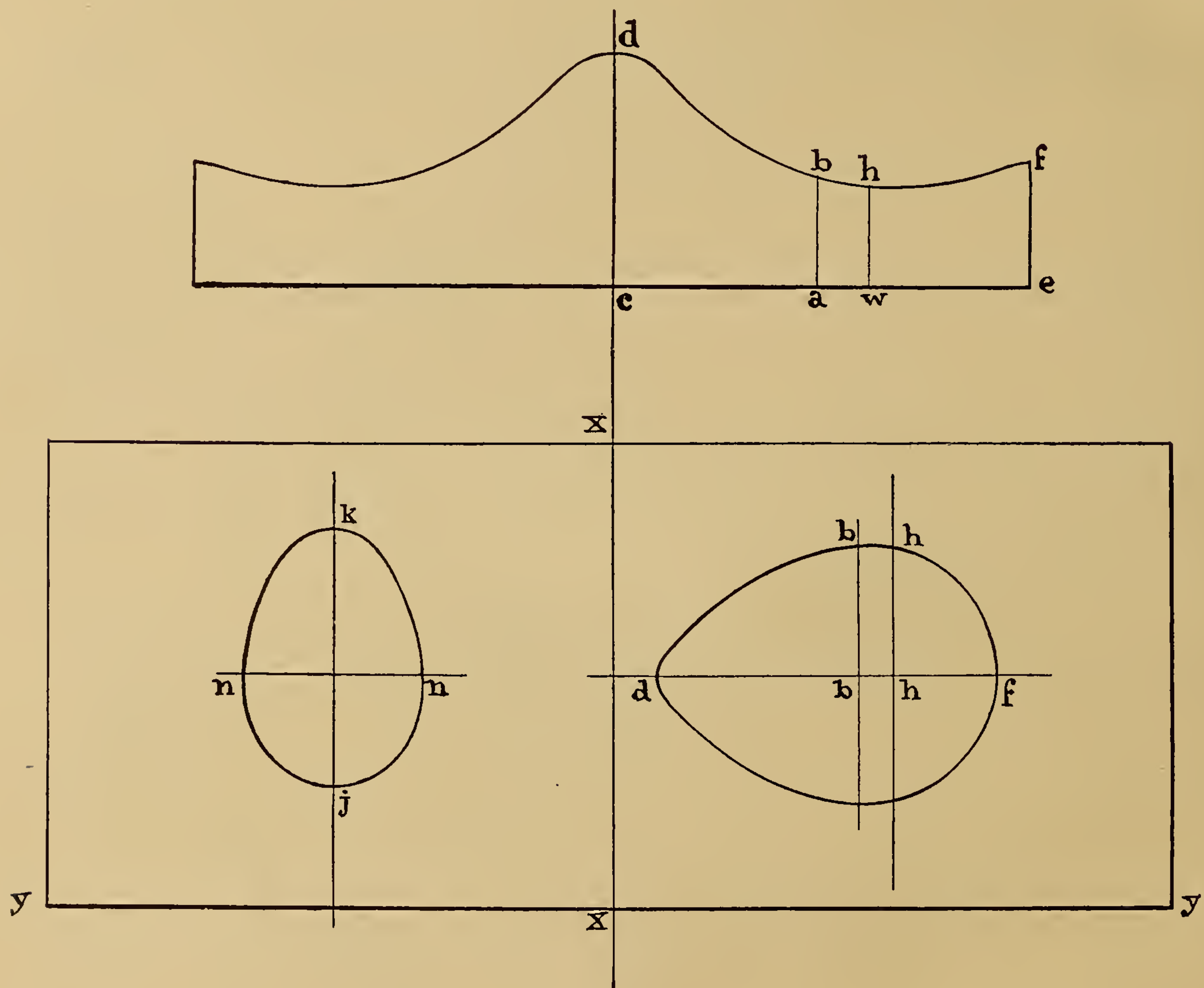


Fig. 116.

end of the sheet, and the remaining part in the other. In the lower part of Fig. 116, then, the development is made on the supposition that the cut is made down the remote element, *y*, in view *C* of Fig. 115, and the cylinder unrolled to the right and left, so that the nearer element *xx* appears as the vertical centre-line. The opening for the horizontal cylinder will then appear on the right, its vertex *d* being at a distance from *xx* equal to the arc *xd*, and its length being equal to the arc *dbf* in view *C*. It will be symmetrical about a horizontal line *df*, at a height above the base equal to that of the axis of the cylinder. The ordinates

of this curve are equal to those of the circle in view *B*: thus, making *db* = arc *dob* in view *C*, the greatest ordinate *bb* is equal to the ordinate at *a*, that is, to the radius; making *bh* = arc *bh* in *C*, set up *hh* = ordinate at *h*, in view *B*, and so on. In making the construction the systematic course previously suggested should be adopted, *df* in the development and the arc *dbf*, being divided into the same number of equal parts, and corresponding ordinates in view *B* being located, just as the one at *l* was from the point *o* in view *C*.

The opening for the inclined cylinder will be on the left, and symmetrical about a

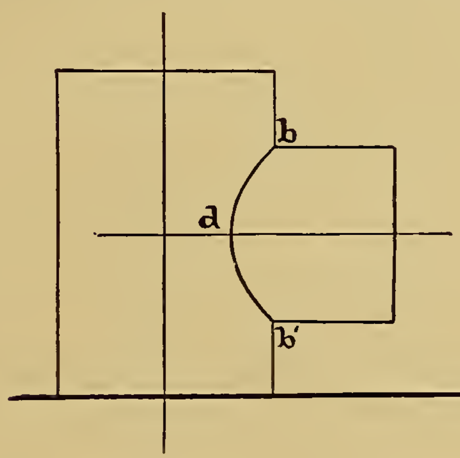


FIG. 117.

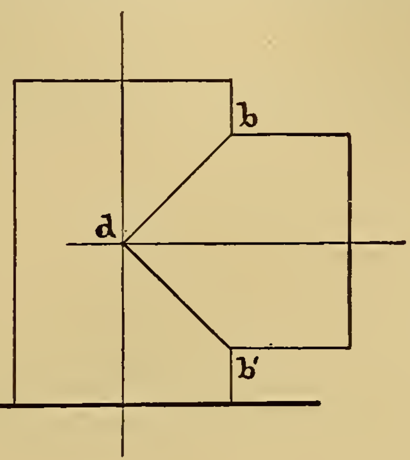


FIG. 118.

vertical line midway between *xx* and the end of the sheet. Its greatest breath *nn* will be equal to the rectified arc *nkn* in view *C*, and the distances of all points in the curve above the base will be the same as in view *A*.

192. If the horizontal axis in Fig. 115 is moved back, nearer to the vertical one, the points *d* and *f* in view *A* will approach each other, *d* moving to the right and *f* to the left, until, when the axes meet, the curves *db* and *fh* will coincide, as shown in Fig. 117; which is a case very often occurring in practice, a familiar example being that of a boiler with a steam-drum. If at the same time the two cylinders are of equal diameters, the intersection will assume the form shown in Fig. 118, being composed of two plane curves *db*, *db′*, each plane making an angle of 45° with either axis; which indeed might have been inferred from consideration of Fig. 103.

If, on the other hand, the horizontal axis be brought forward, the point *d* in Fig. 115 will go to the left, reaching the vertical centre-line when the element *cd* of the horizontal cylinder intersects the element *xx* of the upright one. If the axes be still farther separated, there can no longer be a complete penetration, and the horizontal cylinder must be carried past the other, the effect being shown in Fig. 119. None of these variations involve any new principle, either in determining the intersections themselves or in their development.

193. In Fig. 120 is shown a cone penetrated on the right by a horizontal cylinder, the axes meeting, and on the left by a vertical one, the axes being parallel. In regard to the former, the highest and lowest points, *f* and *g*, of the intersection are seen directly in the front view. Draw in the three views any element *vc* of the cone; in view *B* this is seen to pierce the cylinder at *a* and *b*, which points are projected to view *A*, and thence to view *C*. Draw an element *vq* tangent to the circumference of the cylinder in view *B*; the point of

contact t is an important limit to the curve in the other views, in which also it will be a point of tangency.

Otherwise: Any horizontal plane, as for instance the one through d in view B, will cut an element from the cylinder and a circle from the cone. The circle in this instance has the radius lh, and the element is de; in view C these are seen to intersect in e, which is projected to view A.

So, too, in the case of the vertical cylinder: the horizontal plane through o, for example, cuts from the cone a circle of known radius, which in the top view is seen to pierce the cylinder at i, which is projected to the horizontal through o in view A. Or had an element vr been drawn in view C, piercing the cylinder in i, the same point would have been located

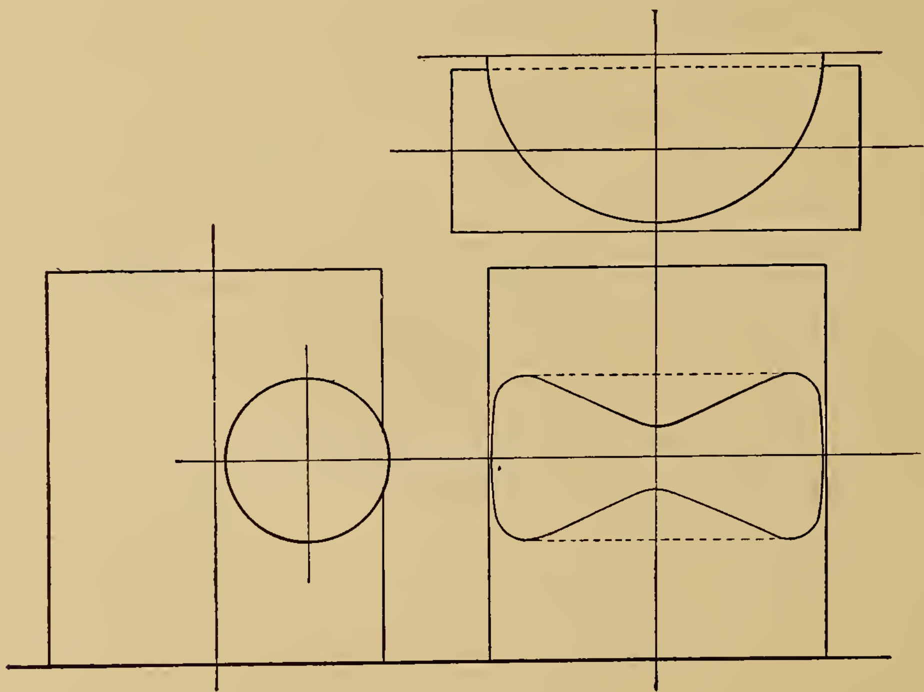


FIG. 119.

by projecting i from C to vr in view A: as it should by this time have been perceived that if a point lies upon any line, its projections must lie upon the corresponding projections of the line. Here too a limit is found by drawing an element vp of the cone tangent to the circumference of the cylinder in view C: the point of contact s is then projected to vp in view A.

194. Let the cone be cut along the remote element, vx' in view B: then in the development, shown at the right, the arcs xx', xx'' are each equal to half the circumference of the base, and are bisected by vm, vn, about which the openings are symmetrical. The arcs mrp, ncq are equal to the corresponding arcs in view C. The vertices f, g are set off by direct measurement of vf, vg, in view A; the points a, b, t are placed at their true distances from the vertex, va', vb', vt' in view B. The developed opening for the vertical cylinder is laid out in a similar manner; in regard to this it may be noted that a check upon the accuracy of the work is found in this, that the length of the arc ii' should be the same in view C and in the development. A system like that previously recommended is applicable to this and all similar cases,—the arcs mp, nq being divided into equal parts on the circle of the base and also in the development, and intermediate elements drawn through the points of division.

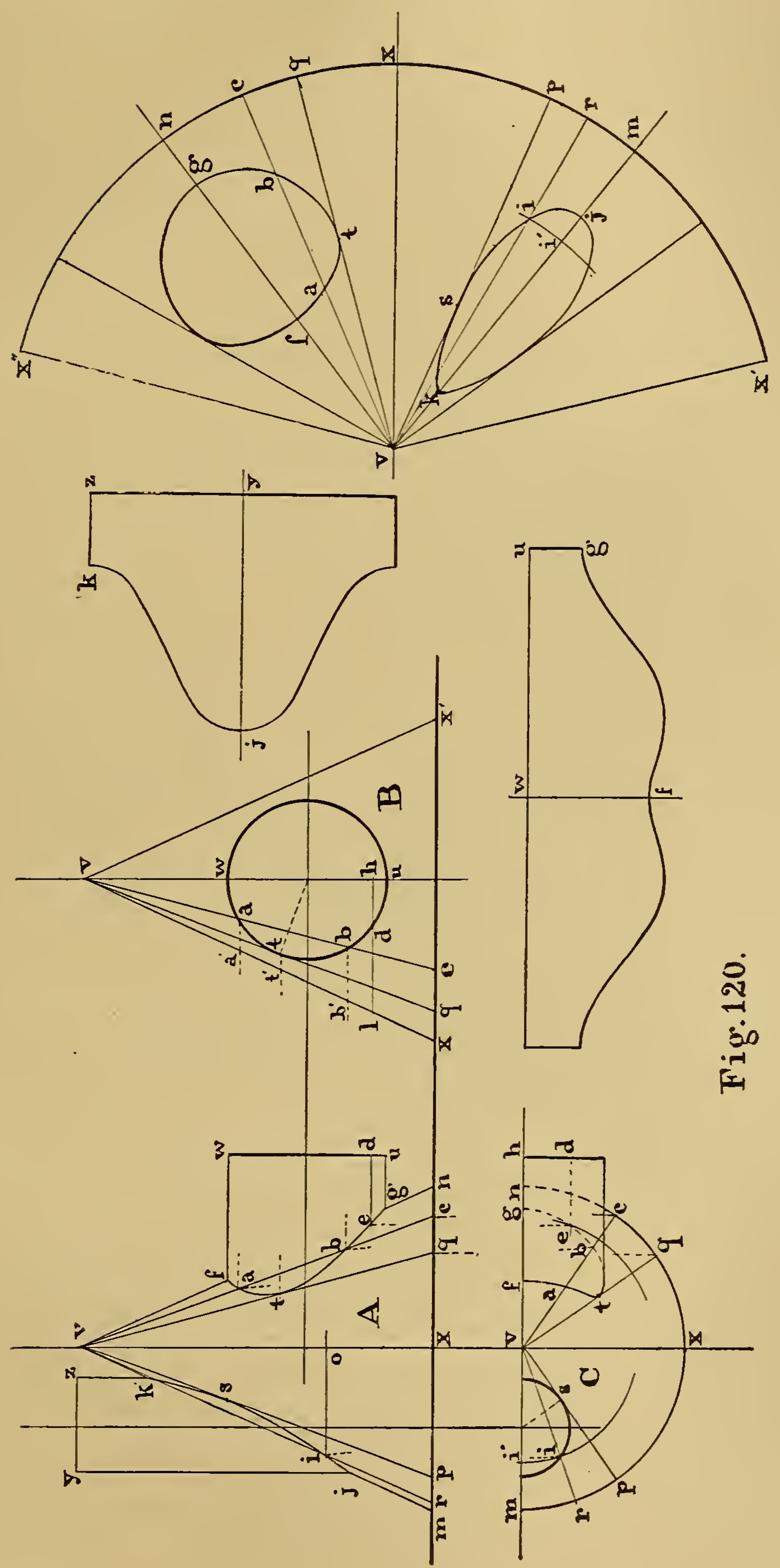


Fig. 120.

The developments of the two cylinders are also shown in Fig. 120; they will be at once recognized, and the construction needs no explanation.

195. In Fig. 121 is represented a vertical cylinder intersected by a cone whose axis is inclined and cuts that of the cylinder; the points *f*, *g* of the intersection are at once seen in view *A*.

Extend the cone to any convenient distance within the cylinder, and limit it by a trans-

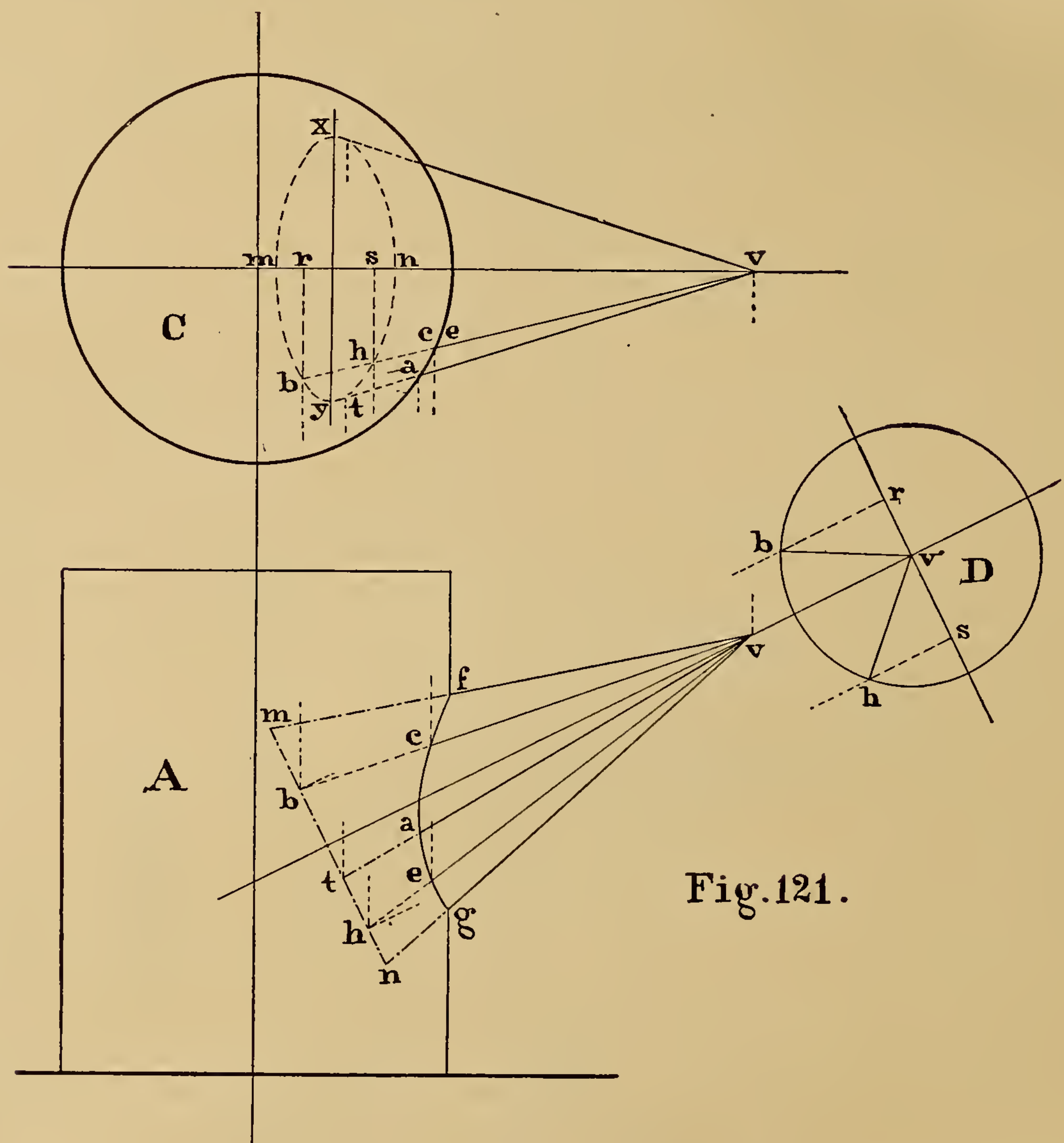


Fig. 121.

verse plane *mn*: the base thus formed is a circle, seen in its true form and size in view *D*, and it is projected as an ellipse in the top view *C*. In this view *C*, the visible contour of the cone is limited by the element *vt* tangent to the ellipse, which pierces the cylinder at *a*. Find the point of tangency *t*, project it to *t* on *mn* in view *A*, in which view draw *vt* and project *a* upon it from view *C*: this gives the left-hand limit of the curve.

Draw any element as *vb* in views *D* and *A*, from the latter project *b* to the ellipse in view *C*; in this view the element thus determined is seen to pierce the cylinder in *c*, which is projected back to *vb* in view *A*. In this way any number of points may be determined; as a

check upon the accuracy, it should be noted that the ordinate *br* is of the same length in views *C* and *D*.

In view *C*, *vb* cuts the ellipse also in *h*, which being projected to *mn* in *A*, shows at once that there is another element *vh* on the lower part of the cone, vertically under *vb*, and piercing the cone at a point *e* vertically under *c*. Or it may be inferred, since the axis of the cylinder is vertical, that a plane through the vertex of the cone, parallel to that axis, cuts two elements from the cone and one from the cylinder; and these lines intersect in points of the required curve.

The process would evidently be the same, were the axes in different planes; common-sense indicating that both should be placed parallel to the paper in the front view. And the

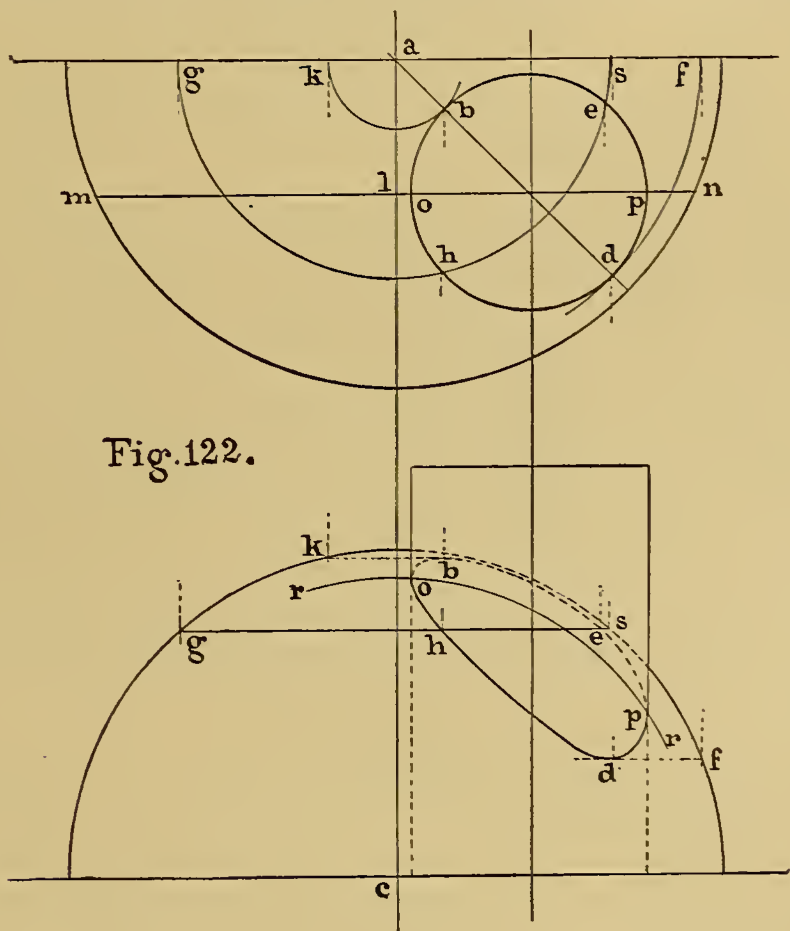


Fig. 122.

development of these surfaces should now present no difficulty to the student, for whose ingenuity the operation is accordingly left as an exercise.

196. Fig. 122 shows a hemisphere penetrated by a vertical cylinder. Any plane parallel to the paper in the front view will cut a circle from the sphere, which in that view will be seen in its true form and size: it will cut the cylinder in two vertical elements if it cuts it at all, and these will intersect the circle in points common to both surfaces. Thus the plane seen edgewise as *mn* in the top view, cuts from the cylinder the extreme right- and left-hand elements, which intersect the circle cut from the sphere, seen as *rr* in the front view, in the points *o* and *p* of the curve of intersection.

Any horizontal plane, as *gs* in the front view, cuts the sphere in a circle, which in the top

view is seen to pierce the cylinder in *h* and *c*, which, projected back to *gs* in the front view, locate two other points in the curve. If in the top view we draw a circle as *df* tangent to the circumference of the cylinder, we can ascertain, by projecting *f* to the outline of the sphere in the front view, the level of the plane by which that circle was cut out, and thus locate *d*, the lowest point of the curve: in like manner the highest point *b* is determined.

197. In Fig. 123 let the two lines *macn*, *oacp* revolve together about the axis *op*. These lines are tangent to each other at *a*, and cut each other at *c*, and will do so throughout the revolution, so that the two circles *ab*, *cd* are common to the two surfaces thus generated.

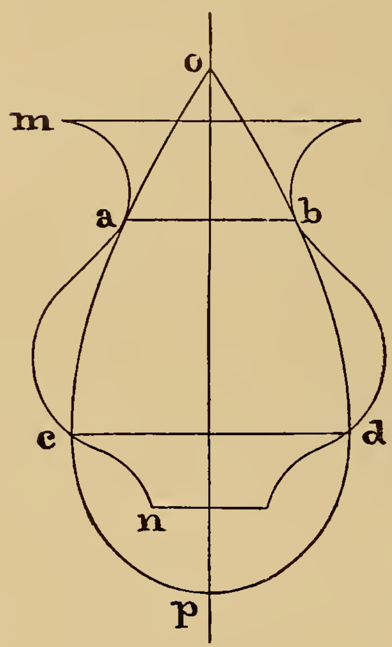


FIG. 123.

Which illustrates a general truth, viz., that if two surfaces of revolution, having a common axis, touch or cut each other at any point, they will do so all round the circle described by that point. In the illustration both the outlines are curved, but clearly either or both might as well have been straight.

The above property may now be applied in finding the intersection of the two surfaces of revolution whose axes intersect, as at *c* in Fig. 124. The simplest lines that can be drawn upon either surface are circles; but if transverse sections are made at random, there is no certainty that the circle thus cut from one surface will intersect the one cut from the other. In order to ensure this, take *c* as the centre of a sphere,—for instance, the one whose outline is *ab*. This cuts from the inclined surface the circle seen edgewise as *oc*; it cuts from the vertical one a horizontal circle through *m*, and the intersection *p* clearly represents a chord common to these two circles, perpendicular to the paper: therefore *p* is in this view a visible point lying on both surfaces, and therefore in the curve sought.

In this case the same sphere serves to locate another point, since *ab* cuts the contour of the vertical surface in *n*, a horizontal through which cuts *oc* in *r*, which also lies upon the required curve. In like manner another sphere whose outline is *gh*, cutting the contour-lines in *k* and *l*, enables us to locate the point *s*; and by repeating this simple operation a sufficient number of times, the intersection of the surfaces may be drawn with great accuracy.

In the ordinary practice of mechanical drawing it is very rarely necessary to construct the top view of an intersection of this nature. Should the occasion arise, however, the student should have no difficulty in doing it: since the radii of the circles through *m*, *n*, etc., are known, it will be seen that the explanations given in connection with Fig. 120 cover the whole ground.

198. In Fig. 125, *v* and *w* are the vertices of two cones of revolution, whose axes, both of which are parallel to the paper in the front view, intersect at *a*. The points *f* and *g* of the curve of intersection are seen by inspection of this view; and other points might be found by means of auxiliary spheres, having *a* for a centre, and cutting circles from each cone, as in the preceding example; but if it is proposed to develop the cones, another method may be preferable.

A plane through the vertex of either cone will cut two elements from it if it cuts it at all; if the same plane passes through both vertices and cuts both cones, the lines cut from one cone will intersect those cut from the other: but any plane through v and w must contain the line vw, upon which the plane turns like a door upon the centre-line of its hinges.

In the application of this it is desirable that the cones should have a common base. The base of the vertical cone is the horizontal plane represented by NN, and is a circle whose diameter is xy. Produce the inclined cone until NN cuts it: the section, or in other words the base of this cone in the same plane, will be an ellipse, whose major axis is lk, and this is

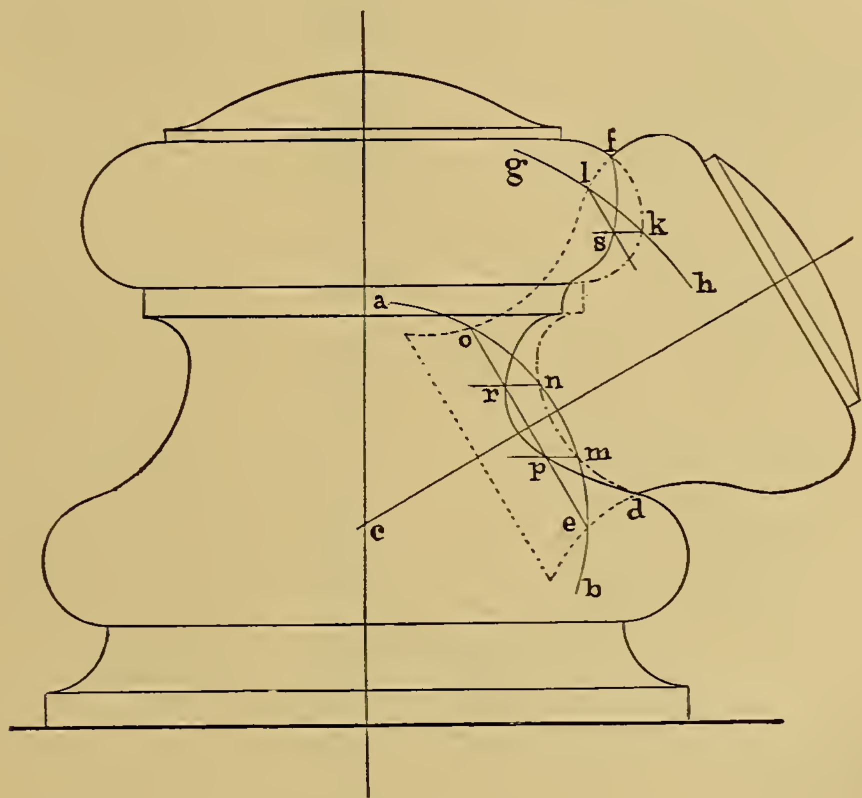


FIG. 124.

readily drawn as in Fig. 105. Produce vw until it pierces the plane NN in the point p. The perspective sketch Z shows a cone whose base is in the plane NN, and a line vp through the vertex, piercing the same plane at p; if in that plane any line pi be drawn, cutting the base of the cone at r and n, it will at once be seen that vp and pi determine a plane which cuts from the cone the two elements vr, vn; also, if pj is drawn tangent to the base at t, the plane vpj is tangent to the cone along the element vt.

199. Now, returning to the principal diagram, draw in the top view the line pn, cutting the elliptical base of the inclined cone in r and n: this determines, as just seen, two elements, rw and nw. But pn cuts the circular base of the vertical cone in o, determining in like manner the element vo, which cuts rw in c and nw in d, and these are two points in the curve of intersection.

A limiting point in this curve is determined by drawing in the top view pt tangent to the ellipse, cutting the circular base at s. Drawing the elements tw, sv, they intersect at b, at which point sv is tangent to the curve, which is clearly shown in the front view: in order

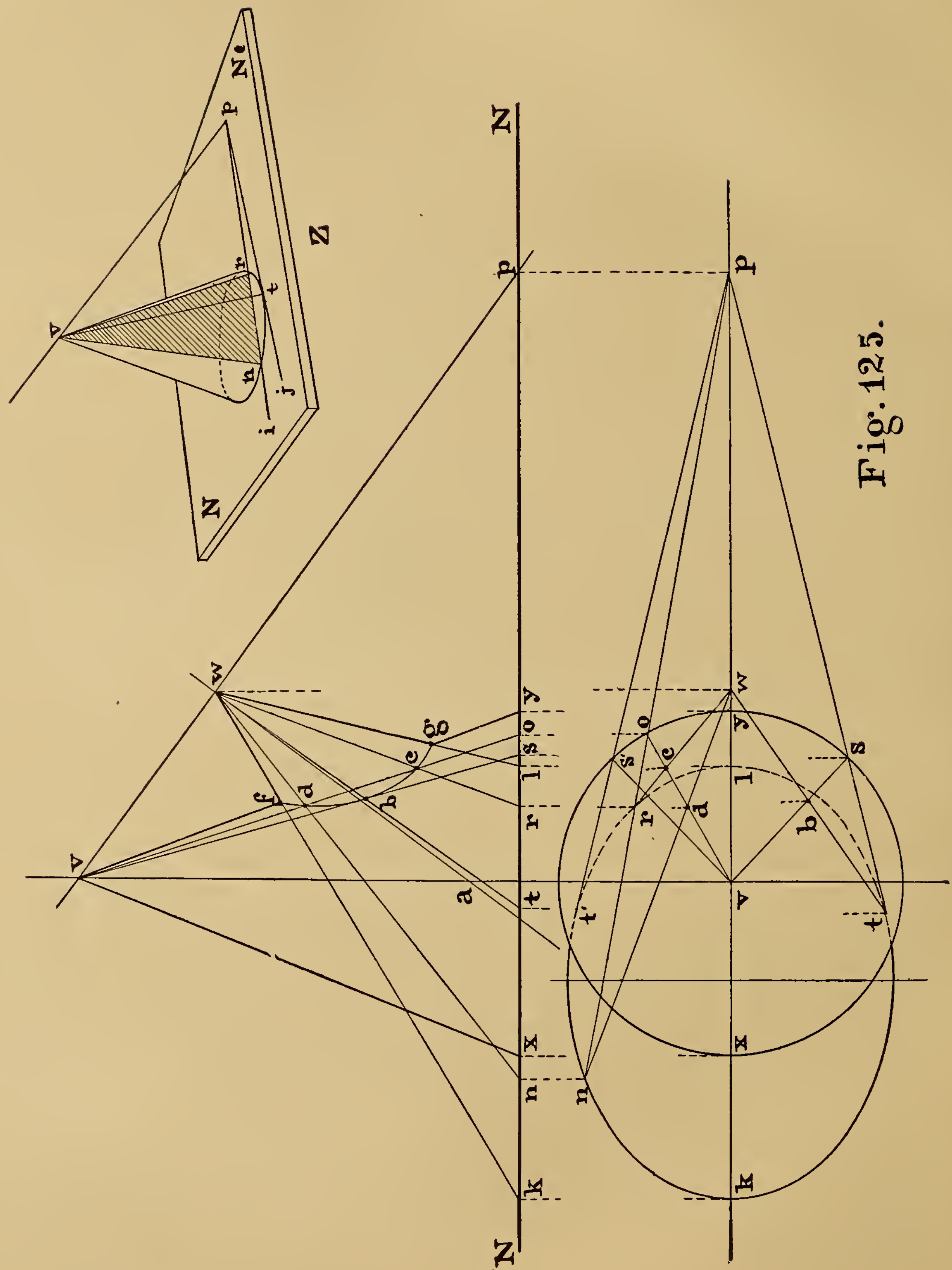


Fig. 125.

to avoid confusion, the curve is not drawn in the top view, the object being to exhibit as clearly as may be the mode by which the points are located.

The process of development, and the use of such limiting lines as *vs*, will be readily understood by referring to the explanation of Fig. 120; it being hardly necessary to add, that a right section of the inclined cone must be made at some convenient point upon its axis, which will develop into a circular arc.

200. Fig. 126 represents the hub of a screw-propeller, having the form of a sphere, with portions cut off by planes perpendicular to the axis at equal distances from the centre *o*: these planes also limit the edges of the blade, *ad* and *ce*. The surface of this blade (here supposed to have no sensible thickness) is the same as that of the square-threaded screw,

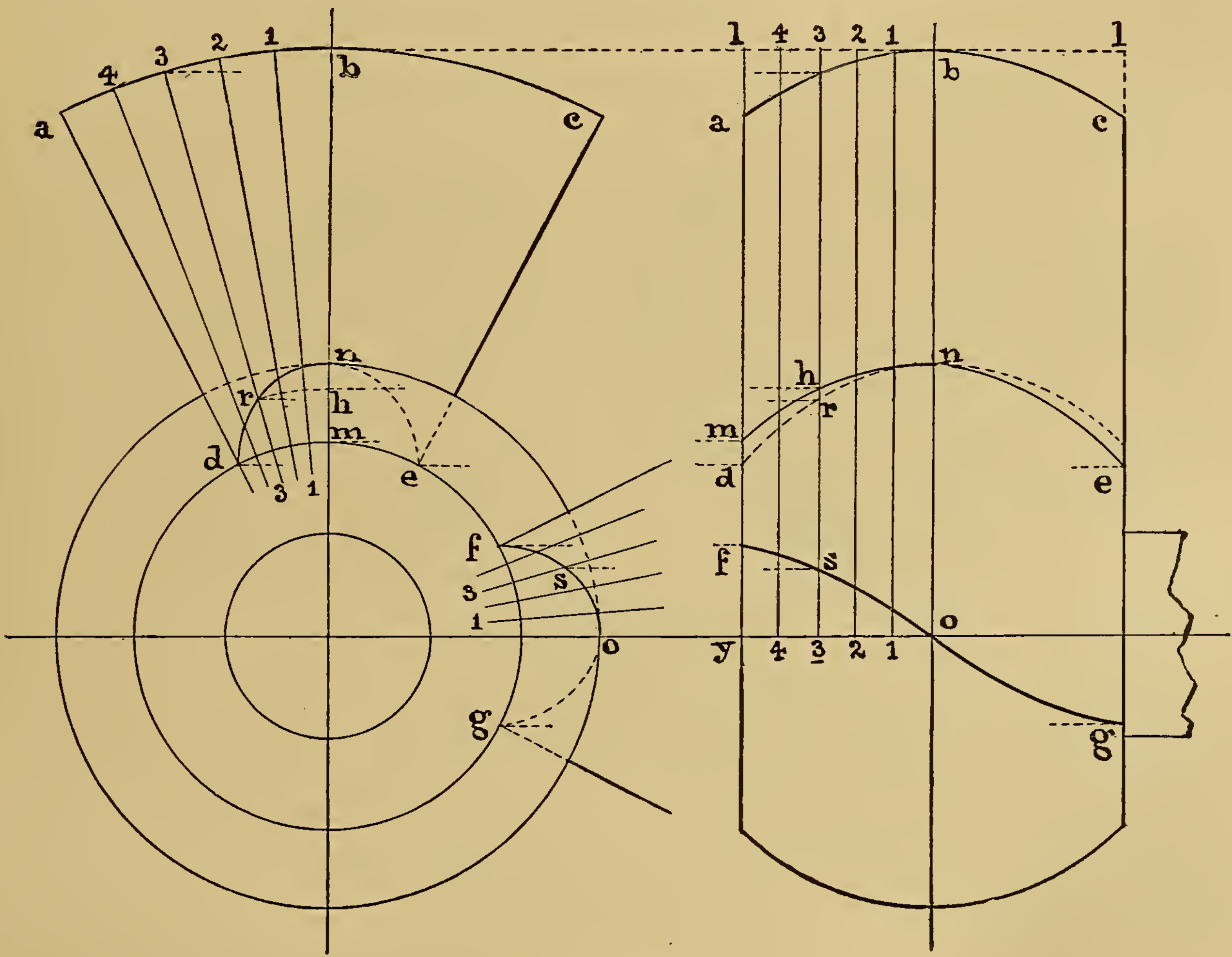


FIG. 126.

described in Chap. V: that is, it is generated by a straight line *bo* perpendicular to the axis, which, while travelling uniformly in the direction *oy*, also revolves uniformly about the axis as indicated by the radial lines in the end view, the point *b* describing the helical arc *ba*. It is now required to find the intersection of this surface with the hub.

If *bo* advanced along the axis without revolving, it would, when in the position $\underline{3}$ 3, pierce the sphere at *h* in the front view; and $h\underline{3}$ is the actual distance from the axis at which it does pierce it. But it has meantime revolved to the radial position $\underline{3}$ 3 in the end view; therefore set off from the centre on this radius a distance equal to $h\underline{3}$, which locates the

point *r*: and in like manner any number of points on the intersection *drne* may be determined.

Which may be expressed by saying that a plane perpendicular to the axis cuts a right line from the blade and a circle from the hub, and the circle cuts the line in a point of the required curve.

If the hub be turned a quarter round toward the right, the curve *dne* appears in the position *fog*; this must be first drawn in the end view, with the radii as shown: the side view is then determined by projecting *f*, which is the new position of *d*, to *my*; *s*, the new position of *r*, to *h*3; and so on.

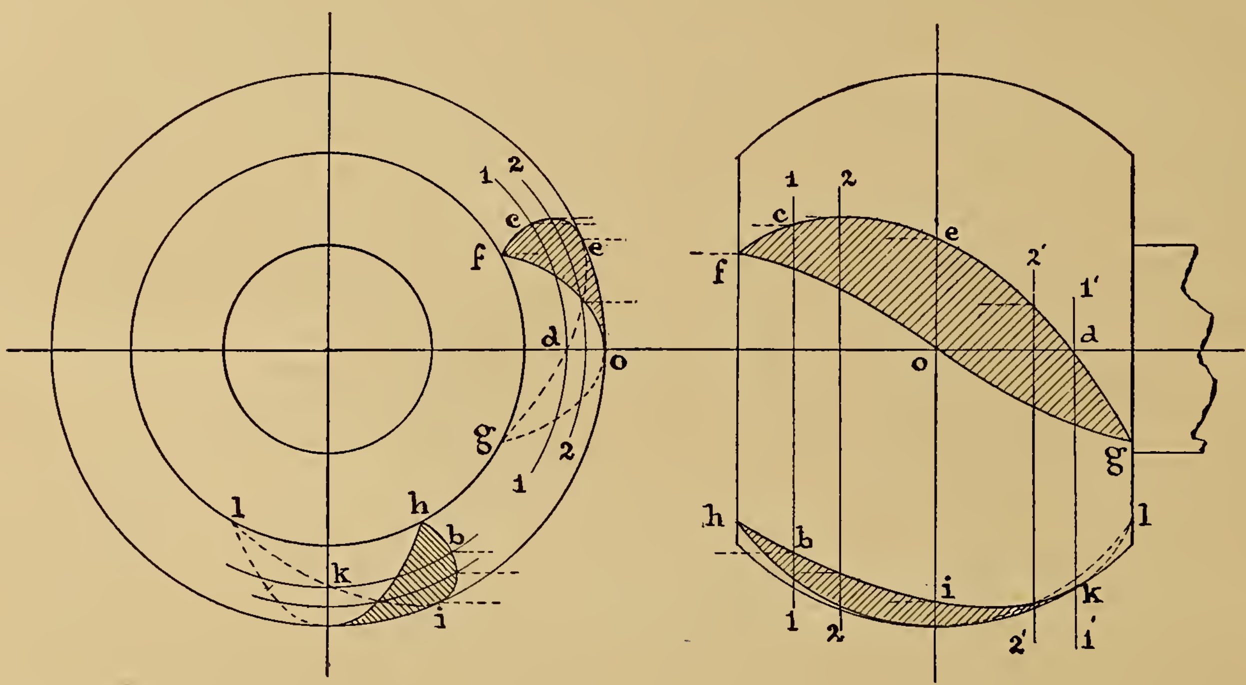


FIG. 127.

201. The blade of a propeller must have sensible thickness; and in forming the back, it is common to assume the curve of intersection with the hub, which determines the thickness of the blade at the root. In Fig. 127, the hub, and the intersection *fog* with the acting face, are the same as in Fig. 126; and *fceg* is the assumed line of junction of the back with the hub, in the front view. To construct the end view, make transverse sections 1 1, 2 2, etc.: these appear in the end view as circles, to which the points *c*, *e*, *d*, and so on, are projected from the front view.

Turning the hub to the right through an angle of 90°, the curve appears as *hbikl* in the end view, and the front view is constructed from this by projecting the various points back to the planes 1 1, 2 2, as in the preceding example.

It may be noted, that by making the sections equidistant from *o* in the front view, as for instance 1 1, 1′ 1′, each pair will be represented by one circle in the end view, which thus serves for the location of two points: thus *c* and *d* lie upon the circular arc 1 1, and so on. This expedient can be employed, of course, only when, as in the example here selected, the outline of the hub is symmetrical with respect to a transverse plane,—which is not by any means always the case in practice.

CHAPTER VII.

ISOMETRICAL DRAWING, CAVALIER PROJECTION, AND PSEUDO-PERSPECTIVE.

ISOMETRY.

202. In Fig. 128, *C* is a top view of a cube so placed that in the front view *A* the diagonals *cg*, *ab* of its upper face are respectively parallel and perpendicular to the paper. The cube is shown as cut by a plane *pp*, perpendicular to the paper in view *A*; the section thus made, as seen in the perspective sketch *V*, is bounded by the three face diagonals *ab*, *ad*, *bd*: it is, then, an equilateral triangle, to the plane of which the three equal edges *ca*, *cb*, *cd* are equally inclined. And as seen in view *A*, this plane is perpendicular to the body diagonal *ch* of the cube, which pierces it at *o*.

In the view *D*, which is an **orthographic** projection upon the plane *pp*, the three face diagonals are seen in their true lengths, forming the equilateral triangle $a'b'd'$. Since the three edges which meet at *c* are equally inclined to the plane, they will be equally foreshortened: therefore c' is the centre of the triangle, $a'c'$, $b'c'$, $d'c'$ are equal to each other, and the three angles at c' are each equal to 120°.

Every other edge of the cube being equal and parallel to one of these three, each visible one will appear equal and parallel to one of those already drawn; thus the apparent contour of the entire cube will be a regular hexagon, the representation of each face being a rhombus.

Because the edges of the cube are thus foreshortened in the same proportion, so that they and all parallels to them may be measured by the same scale, such a view as *D* is called an **Isometric Projection**; $a'c'$, $b'c'$, $d'c'$ are called the *isometric axes;* the planes which they determine, and all planes parallel to them, are called *isometric planes;* and all lines parallel to the axes are called *isometric lines.*

203. Drawings made in this manner possess the advantage of conveying, in one view, ideas of the three dimensions, as do those made in perspective; and in many cases they exhibit the peculiarities of structure more clearly than ordinary plans, sections, and elevations. They are readily understood by those who are not familiar with common projections; and in making sketches this system is very useful.

Obviously, however, the advantages of isometry are more pronounced when the objects to be represented are bounded by right lines, of which the principal ones are parallel and perpendicular to each other. It is not well adapted for the general drawing of machinery, since it involves an unpleasant distortion, and also because in most cases the circles are projected as ellipses.

204. *Distinction between isometrical* **projection** *and isometrical* **drawing.**—In Fig. 128 the actual length of the edge of the cube is *cd*; its apparent length in view *D* is $c'd'$, equal to *od* in view *A*. Suppose *cd* to be one unit in length—an inch for example: then by taking *od*

as a unit it is possible to construct an *isometric scale*, by which all the isometric lines in *D* might have been set off; and such a scale could be used in constructing any isometrical *projection*.

This is a matter of purely abstract, theoretical interest, and not of any practical use whatever. Since the isometric lines are all *equally* foreshortened, there is no reason why they should be represented as foreshortened at all. Consequently an **Isometric Drawing** of the given cube is made as shown at *E*, each edge being drawn of its true length. This is the method always adopted in practice, the scales in common use being alone employed. The man who should construct a true *projection*, and send it to the workman to be measured, by

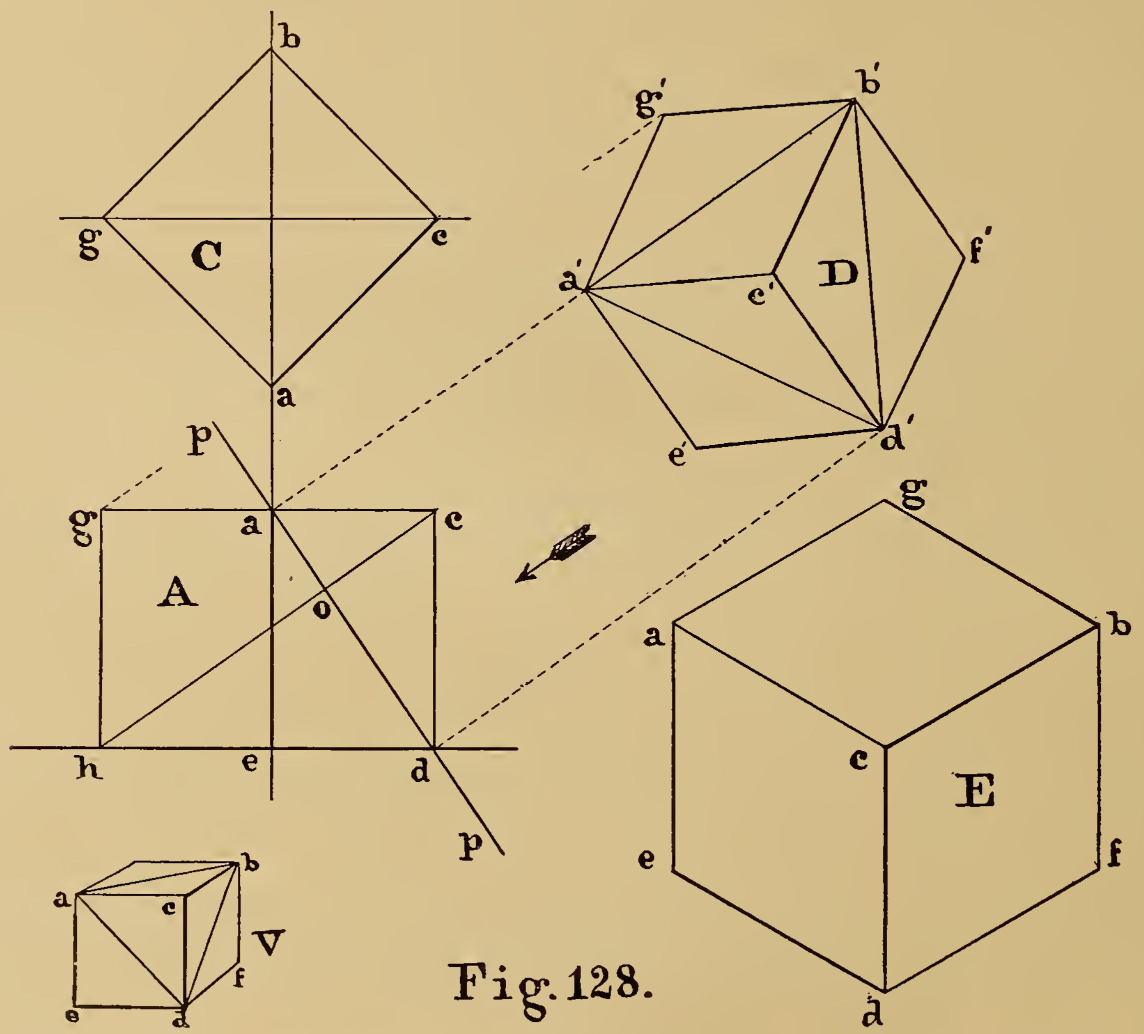


Fig. 128.

an isometric scale, would simply make a record of his own stupidity; he who should teach others to do so, would commit a blunder of much more serious importance. For, to use the words of another, "the value of isometry as a practical art lies in the applicability of common and known scales to the isometric lines."*

205. We proceed, then, just as in making ordinary working drawings, setting off the dimensions on those lines either "full size," or with the 3-inch scale, the $1\frac{1}{2}$-inch scale, etc., as the case may require. Naturally, lines which are vertical are so represented; the other

* W. E. Worthen.

isometric lines are then drawn with great facility by the aid of the T-square and the triangle of 60° and 30°.

Figs. 129–134 are simple exercises, composed wholly of isometric lines, the construction being so obvious that no detailed explanation is required: the method of locating the foot of the cross in Fig. 132, and the mortise and the tenon in Fig. 134, by measuring along the lines *ca*, *cb*, or parallels to them, is sufficiently shown by the dotted lines.

It is to be distinctly understood that these figures are illustrations merely: the student is not to *copy* them, but to construct them or others of similar character, with such variations of dimensions, arrangement, or design as may be suggested by his ingenuity, which should be given full play.

In regard to the shadow-lines, reference is made to a cube as a standard. Thus in *E*, Fig. 128, the light is supposed to have the direction of the body diagonal *af*; consequently

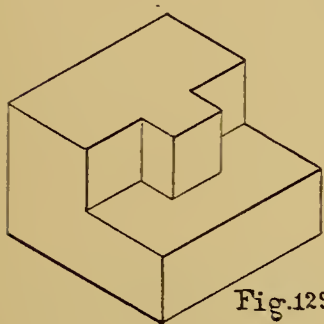


Fig. 129.

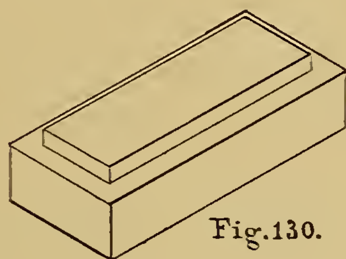


Fig. 130.

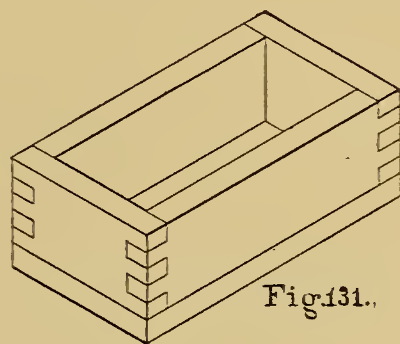


Fig. 131.

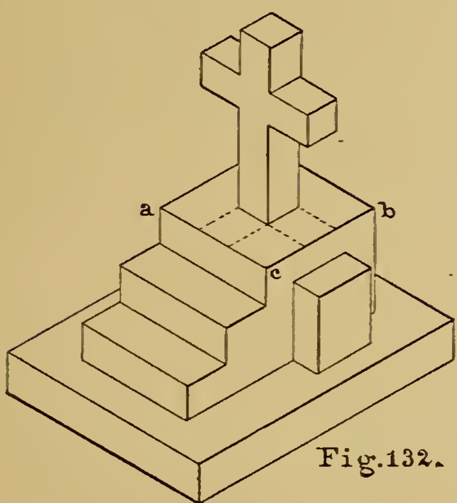


Fig. 132.

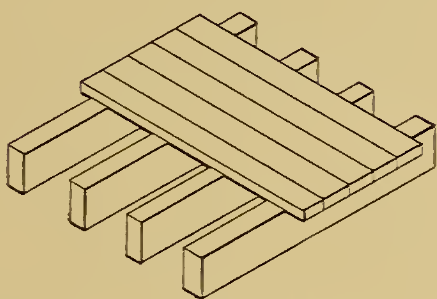


Fig. 133.

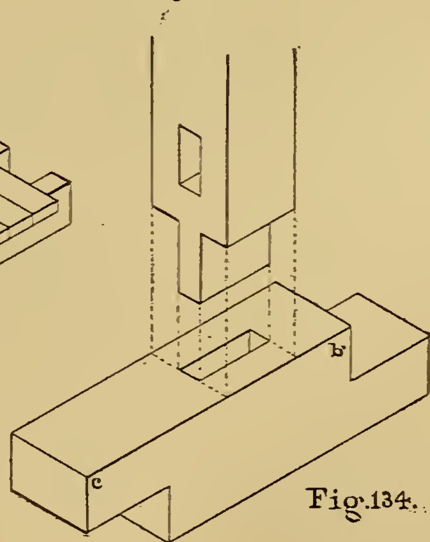


Fig. 134.

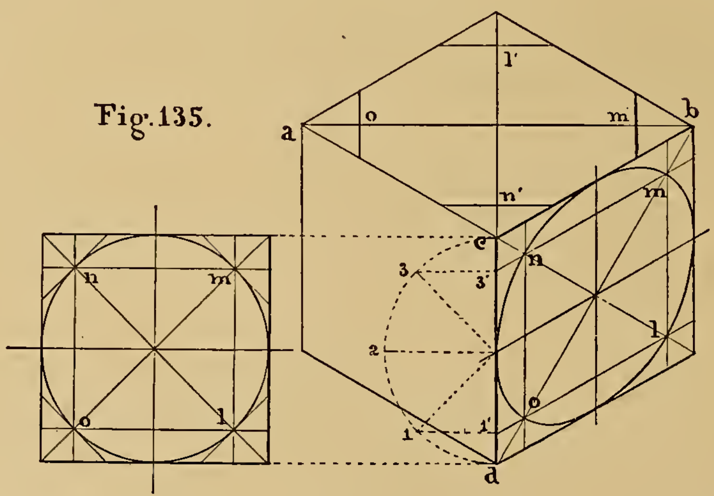
the faces *cc*, *cg* are illuminated, and shadows are cast by the edges *ed*, *dc*, *cb*, and *bg*. In drawing, the first of these lines should be made the heaviest, the last one the lightest, and the other two of equal and medium thickness.

206. Isometrical Drawing of the Circle.—The ellipse representing the circle inscribed in the face of the cube, Fig. 135, might be drawn by the method of Fig. 40 (Chap. II). But the axes coincide with the diagonals, and are at once determined by representing the parallels through *l*, *m*, *n*, *o*, in the elevation shown at the left. Describe a semicircle upon *cd* as a diameter, divide it into four equal parts by the points 1, 2, 3, draw 3 3′, 1 1′ perpendicular

to *cd*, and through 3′ and 1′ draw parallels to *bc*; these will cut the diagonals at *m*, *o*, and *n*, *l*, thus limiting the major and minor axes. As a check, note that *lm* and *on* should be parallel to *cd*.

The sides of the rhombus being equal, this construction may be made upon either one at pleasure. And, since all the faces of the cube are exactly alike, it follows that all circles lying in isometric planes are represented by similar ellipses.

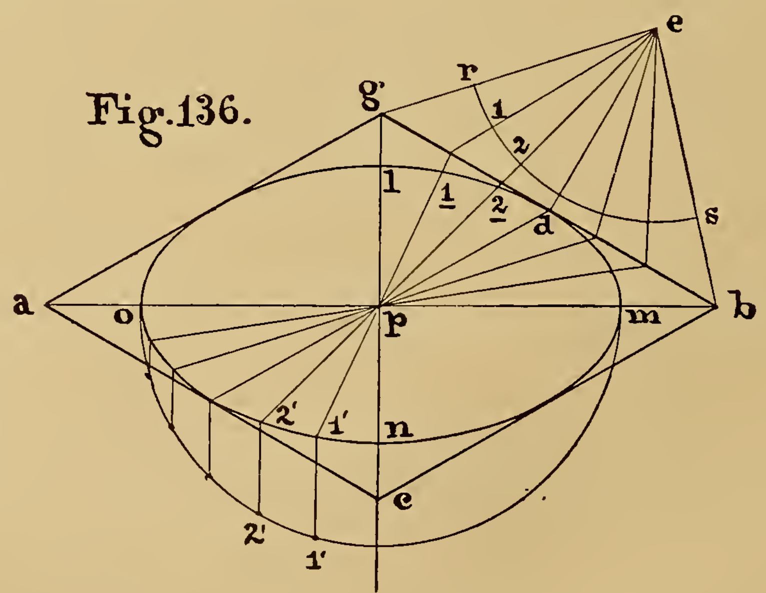
Fig. 135.



By drawing tangents at the points *l*, *m*, *n*, *o* in the elevation the circle is circumscribed by a regular octagon, the isometric representation of which is therefore made by drawing at the corresponding points *l′*, *m′*, *n′*, *o′*, in the upper face of the cube, perpendiculars to the diagonals, terminating in the sides of the rhombus.

207. Graduation of the Isometric Circle.—*First Method.* At the middle point *d* of *gb*, in Fig. 136, erect a perpendicular *de* equal to *db*, and draw *eb*, *eg*; about *e* as a centre

Fig. 136.



describe with any radius the quadrant *rs*, divide it as desired by the points 1, 2, 3, etc., through which draw radii and produce them to cut *gb*. From these intersections with *gb* draw lines to *p*, the centre of the ellipse: these will cut its circumference in the required points of division, 1̲, 2̲, etc.

Second Method. Describe a semicircle upon the major axis mo as a diameter, divide it in the desired manner by the points $1'$, $2'$, etc., through which draw perpendiculars to mo, cutting the circumference of the ellipse in $1''$, $2''$, etc.: these will be the points required.

An application of the above is shown in the drawing of the bolt, nut, and washer, Fig. 137. About p, the centre of the outer ellipse, describe an arc with radius po = semi-major

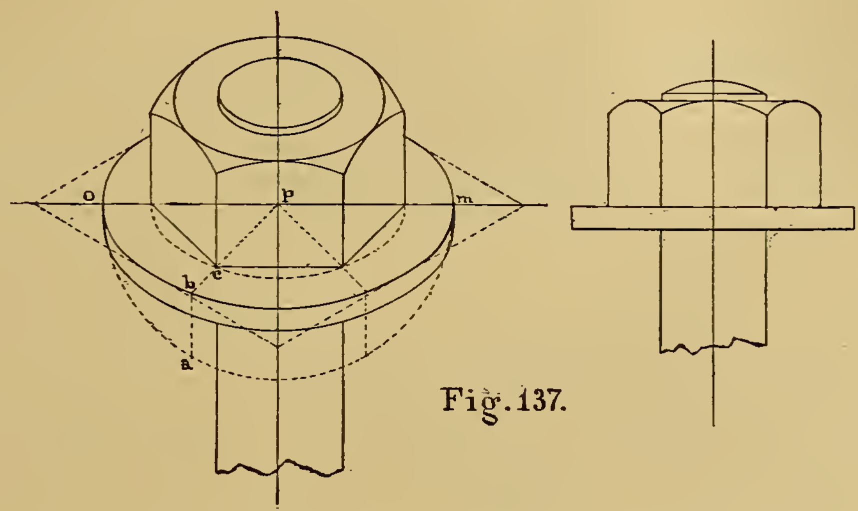


Fig. 137.

axis, set off the arc $oa = 60°$, erect the vertical ab, and draw bp cutting the inner ellipse (circumscribing the base of the nut) in c.

208. To draw Angles to the Sides of the Isometrical Cube. (Fig. 138.)—Draw a square cg, whose side is equal to the edge of the cube; about one of its angles, say c, as a centre, describe the quadrant ab, graduate it, and produce the radii through the points of division to cut the sides of the square. The scale of tangents thus formed may, by cutting out the square, be applied to any side of the isometrical cube, thus determining the direction

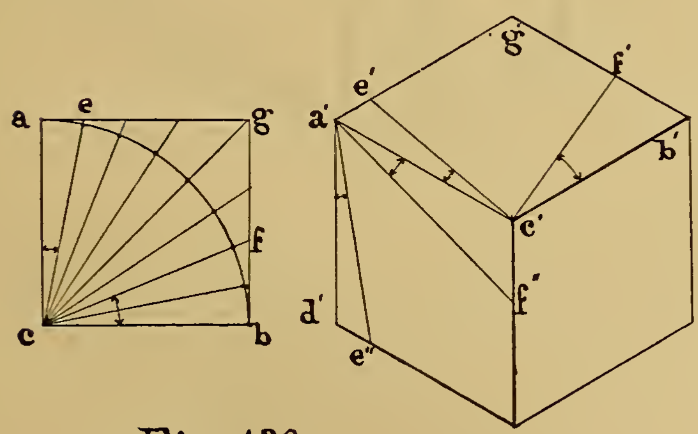


Fig. 138.

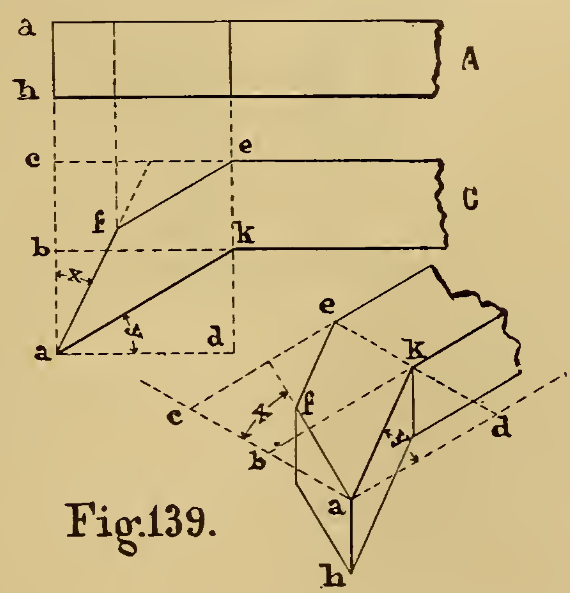


Fig. 139.

of a line in its face which shall represent a line making any required angle with its edge. For example, make $a'e' = ae$, and $b'f' = bf$: then $a'c'e'$, $b'c'f'$ are the isometrical representa-

tions of the angles *ace*, *bcf*. The same angles are represented on the left-hand vertical face of the cube by making $d'e'' = ac$, $c'f'' = bf$, and drawing $a'e''$, $a'f''$.

An application of this is found in making the isometrical drawing of the piece shown in plan and elevation at *C* and *A*, Fig. 139, in which the angles x, y are assigned: also in *C* the distance *ad* is given, *dkc* is perpendicular to *ad*, and *cf* parallel to *ak*: the thickness is uniform, and equal to *ah* in view *A*. The isometric drawing is lettered to correspond, and should require no further explanation.

209. Another method of dealing with lines which, though lying in isometric planes, are not parallel to either of the isometric axes is by means of "offsets." Thus in Fig. 140 the slope of the diagonal brace is determined by measuring the distances *cd*, *dc* along the isometric line *ca*, and setting up the vertical *ef*, of the values ascertained from the elevation

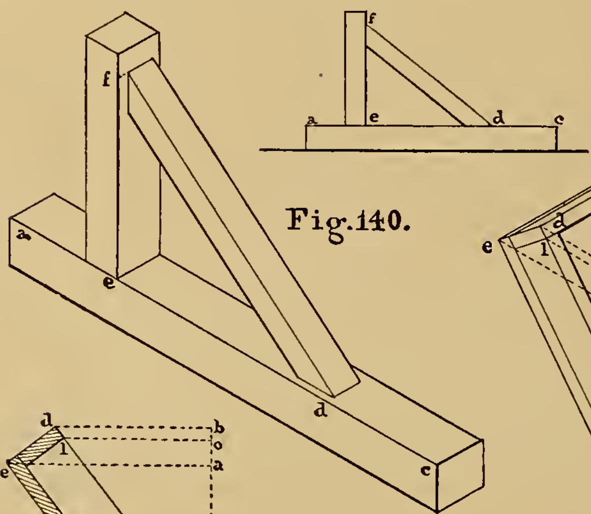
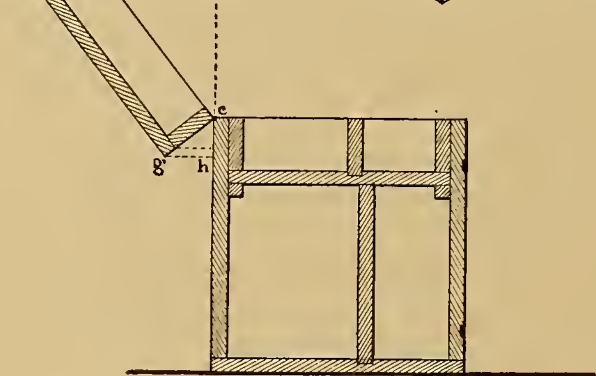


Fig.140.



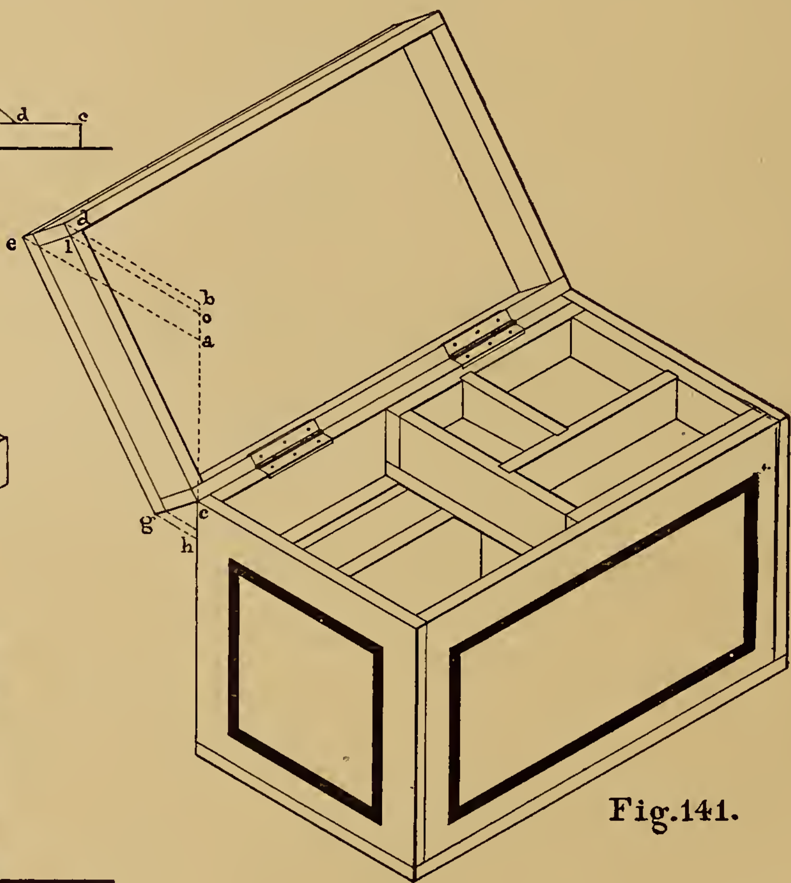


Fig.141.

shown on a reduced scale. This really amounts to the same thing, the angle being constructed by laying off the base and altitude of a triangle of which the required line is the hypothenuse,—which is in perhaps the majority of cases the most convenient means. Another illustration is given in the drawing of the box, Fig. 141; the outline of the end of the partially opened lid being set out by means of the vertical measurements *ca*, *co*, *cb*, *ch*, and the offsets *ac*, *ol*, *bd*, *hg*, taken directly from the transverse section shown at the left.

210. This principle may be extended, and is applied to the determination of lines which do not lie in isometric planes; as illustrated in Fig. 142, representing the roof of a cottage, of the form and proportions shown in plan and elevations on a smaller scale at the right.

The sloping lines of the roof at the nearer end are found by setting up the heights *ab*, *ad* on the vertical through *a*, and drawing the isometric lines *bh*, *df*: then the points *i*, *k* are the intersections of *bf*, *fh* with the isometrical line through *c*. A similar construction may be made at the farther end, thus fixing the line of the ridge *ff′*, on which the point *g* is located by setting off *fg*, its distance from the plane *de*: we can then draw *gi*, *gk*, which do not lie in any isometric plane.

The same process is applied in drawing the wing roof, the heights *n*, *v*, *r* being set up on the vertical through the nearer corner *m*, and the distance *oq* measured from the plane *mo*. The ridge line will pierce the main roof at a point *u*, which may be thus located: Set up *at* = *mr*, draw the isometric line *ts* cutting *bf* in *s*, and through *s* draw a parallel to *ff′*: this will cut the ridge line of the wing in the required point.

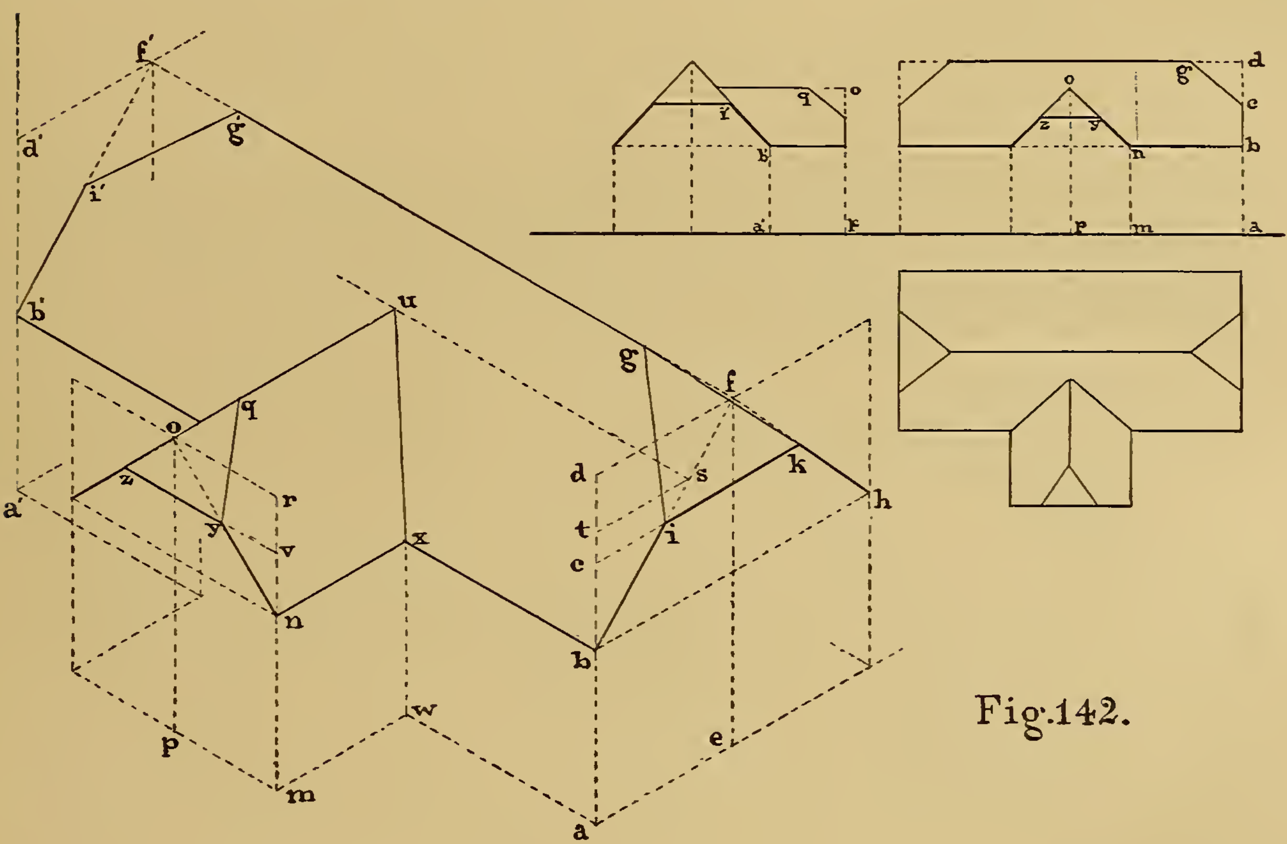


Fig.142.

It will be observed that the lines *fh*, *gk*, and *ff′*, differ very little in direction, and *qz*, *ou* differ still less. This simply shows that the farther side of the main roof is very nearly, and that of the wing roof almost exactly, perpendicular to the plane upon which the isometric drawing is made: and it will be perceived that in such cases this is not a peculiarly eligible mode of representation,—as indeed it is not for architectural subjects of any description.

211. Thus far one of the isometric axes has been made vertical. But inasmuch as it is the relative direction of the lines among themselves which determines whether a drawing is an isometric one or not, there is no necessity that any of them should be vertical. In Fig. 143, for example, the principal lines are horizontal; but the drawings of the die and its

matrix, and of the timber with its mortises and its tenon, are at once recognized as isometric, and are just as easily understood as if they stood upright.

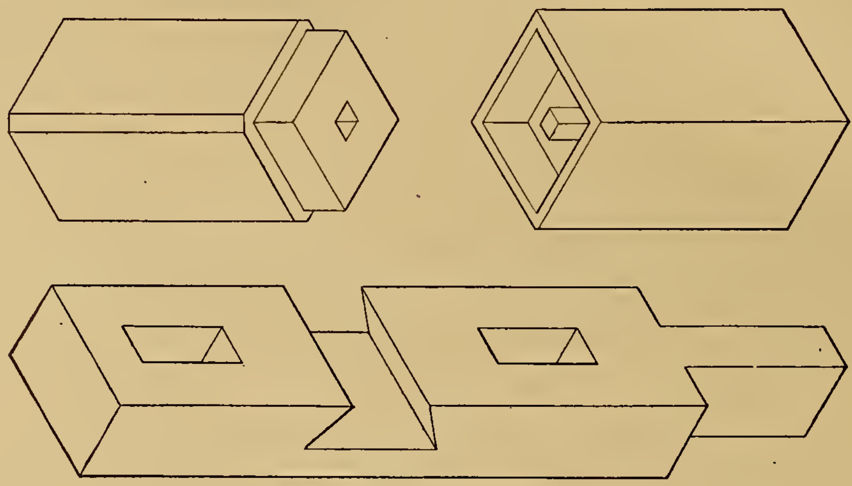
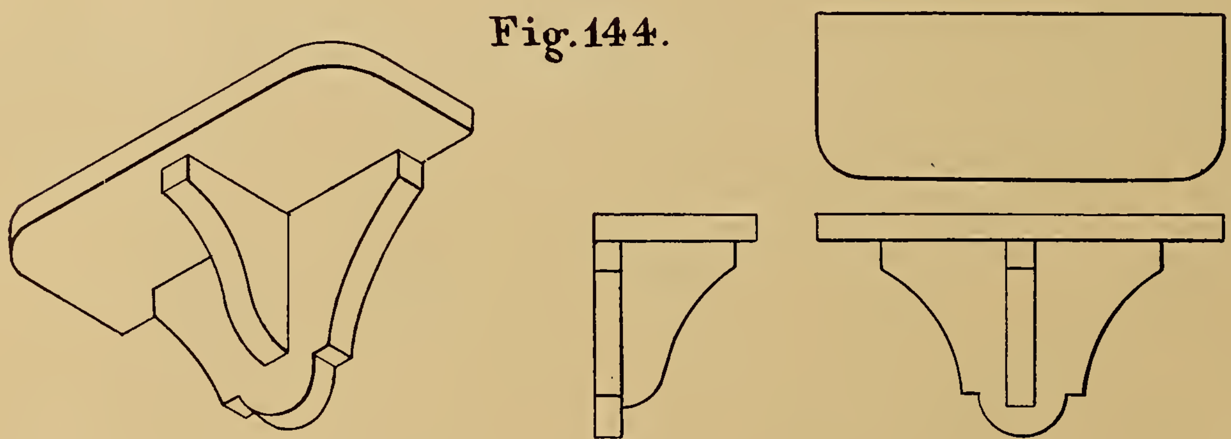


FIG. 143.

For convenience in constructing the drawings by means of the T-square and triangles, it is preferable in most cases, of course, that one of the isometric axes should be either vertical or horizontal, but should there be any reason for selecting other positions, there is nothing in the principle of isometry to prevent their adoption.

Fig. 144.



212. It will be noted that the correspondence of the die to the matrix in Fig. 143 is made much more obvious than it otherwise would be, by exhibiting the opposite ends of the two pieces. By merely turning the page around, it will be apparent that this could have been done equally well if the two pieces had been drawn in a vertical position.

For this reason isometry affords a means of illustrating in a very clear and striking manner many subjects in which views of the lower surfaces are desirable: a good example is shown in the drawing of the small shelf with its supporting bracket, Fig. 144.

In making such a drawing, as will readily be seen, the process is equivalent to constructing the projection of the cube, Fig. 128, upon the plane pp, as seen from the lower left-hand side, and looking in the direction opposite to that indicated by the arrow.

213. In Fig. 145 is shown an isometric drawing of the ratchet-wheel represented in the full size views at the right. The backs of the teeth not only terminate in, but are tangent to, the interior circle; and a test of the accuracy of the isometric drawing is found in the tangency of the edges to the ellipse representing that circle. And this embodies a principle capable of many other applications—as, for instance, in laying out a wheel with radial tapering arms: the side outlines of each arm are tangent to a circle, which being drawn in the isometric construction, it is seen that the breadths of the arms at the outer ends only need be set out, thus fixing points through which tangents are to be drawn to the ellipse.

214. It seems needless to multiply examples, as it is believed that by the aid of the preceding any isometrical drawing likely to be required in practice may be constructed.

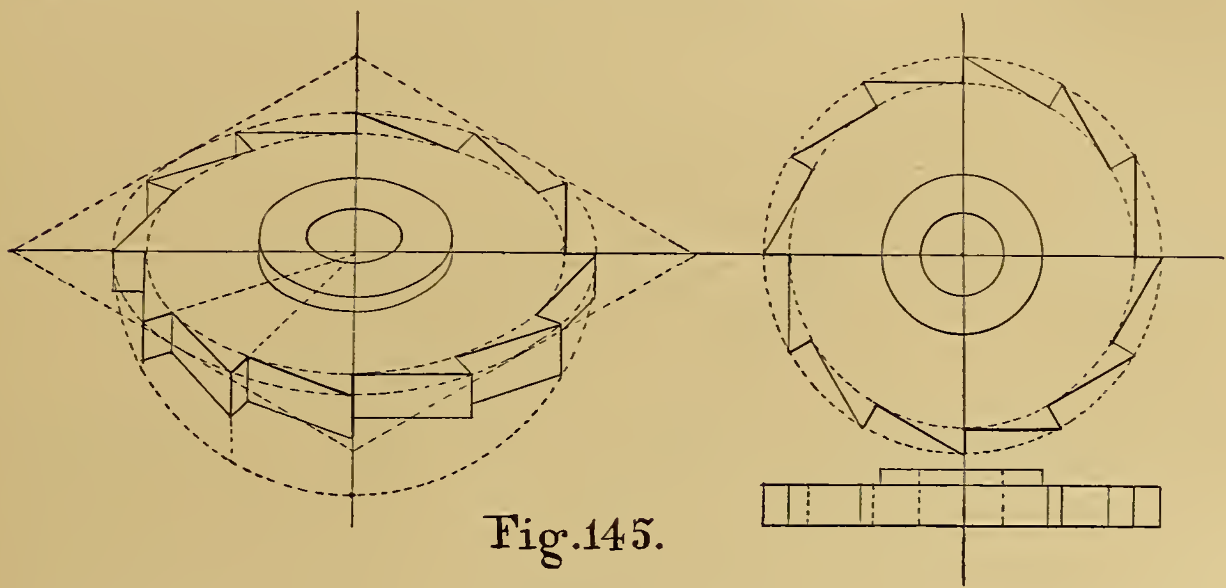


Fig. 145.

In drawings of machinery, the circles of wheels, bearings, ends of shafts, and the like, will usually lie in isometric planes. Should occasion arise to represent one which does not, circumscribe it by a square: the projection of this will be a parallelogram, within which the ellipse may be drawn by methods previously explained. So, too, if it should be necessary to represent the section of a cylinder or a cone by an oblique plane, the solid may be conceived as surrounded by a square pyramid or prism, whose section by the given plane, as well as the isometric drawing of it, will be a parallelogram circumscribing the required ellipse.

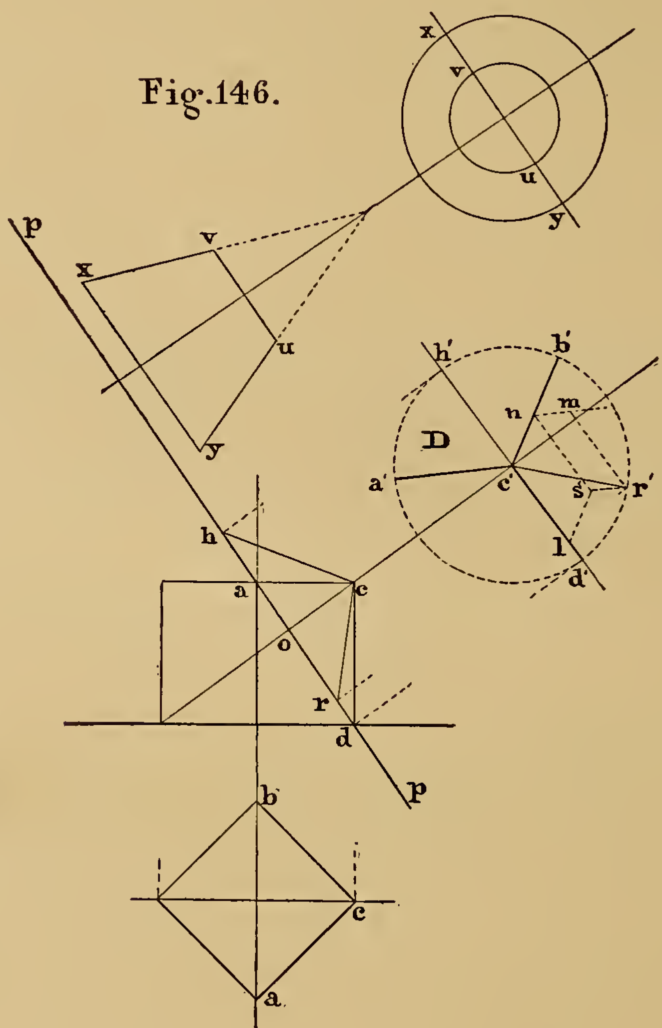
215. In conclusion, it may be pointed out that many lines not usually classed as *isometric* are strictly so in fact. This distinctive term is technically restricted to lines parallel to the isometric axes, which again are so called because they are equally foreshortened, and this is the result of their equal inclination to the plane upon which they are projected. Now, in Fig. 146 the cube is cut by the plane pp as in Fig. 128, and in the projection D we at once recognize $c'a'$, $c'b'$, $c'd'$ as the isometric axes. If cd revolve around co as an axis it will generate a cone dch, all of whose elements make the same angle with the plane pp; so that any one of them, as cr (seen in D as $c'r'$), would be foreshortened in the same proportion as any other one. But in the isometric projection this fact would not be indicated by merely drawing $c'r'$: it is necessary to locate the point r' by means of offsets—$c'n$ giving its distance from the plane $a'c'd'$, $c'l$ its distance from the plane $a'c'b'$, and nm its distance from the plane $b'c'd'$.

This being done, s' is at once seen to be the foot of the perpendicular $r's'$ from the point in question to the plane last mentioned.

Again, the isometric projection of any frustum of a cone, $xyuv$, whose bases are parallel to pp, would appear simply as two concentric circles, and without some auxiliary view that projection would convey no definite information about the cone, which might be of any altitude or have either base uppermost.

Since the whole value of isometry, in practice, lies in the power of imparting in one view

Fig. 146.



definite ideas of the three dimensions, the above hints may serve a purpose as indicating possible relations of parts for the representation of which this method of drawing is not suitable.

CAVALIER PROJECTION.

216. In Fig. 147, let MN represent a vertical glass plate (corresponding to one side of the show-case, Fig. 68); let c be a point in this plane, and ca a line perpendicular to it. Let ar be a visual ray, making an angle of 45° with the plane MN, and piercing it at p: then cp is the representation of ca upon the picture plane, and it is equal to ca, because the angles cpa, cap are each equal to 45°.

Suppose the eye to be at an infinite distance in the direction ar: then all the visual rays

will be parallel, and all lines perpendicular to MN will be represented upon that plane by lines of their actual length, and parallel to cp.

The fact that the projection is of the same length as the perpendicular line depends upon the condition that the picture plane cuts the projecting lines at an angle of 45°. But the direction of the projection depends upon that of the visual ray. Thus if the eye be still at

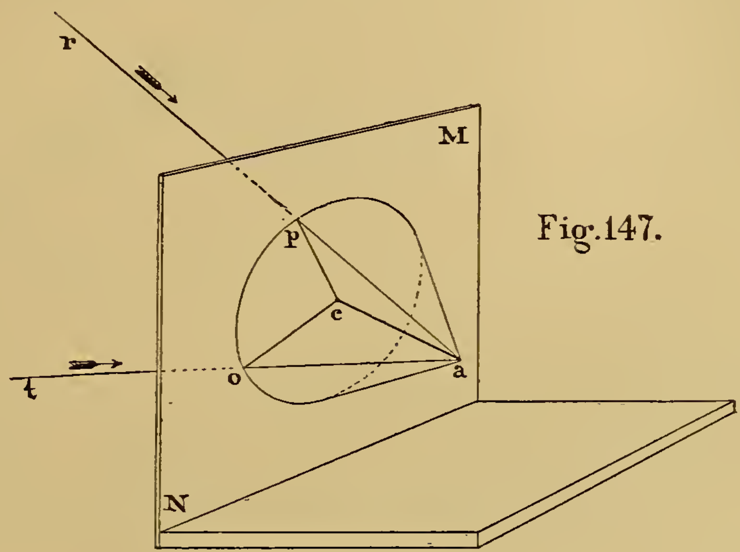


Fig. 147.

an infinite distance, but in the direction at, the projection will have the direction co, but its length will remain equal to ca.

Thus the direction of the projecting lines may be parallel to any element of the cone whose axis is ca, the angle at the vertex a being 90°, since all these elements make angles of 45° with the picture plane MN.

217. Now any line which lies in the picture plane is its own projection. In representing a cube, therefore, as in Fig. 148, we may assume its nearer face to lie in that plane, and it will thus appear of its true form and size, that is, a square, as shown. From the preceding it follows at once that the edges which are perpendicular to MN may be represented by parallel

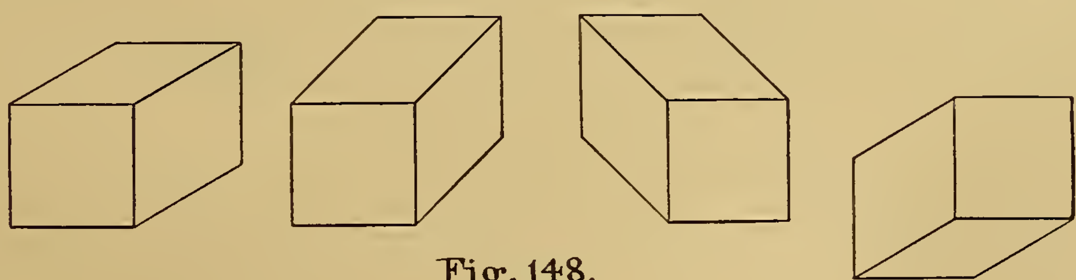


Fig. 148.

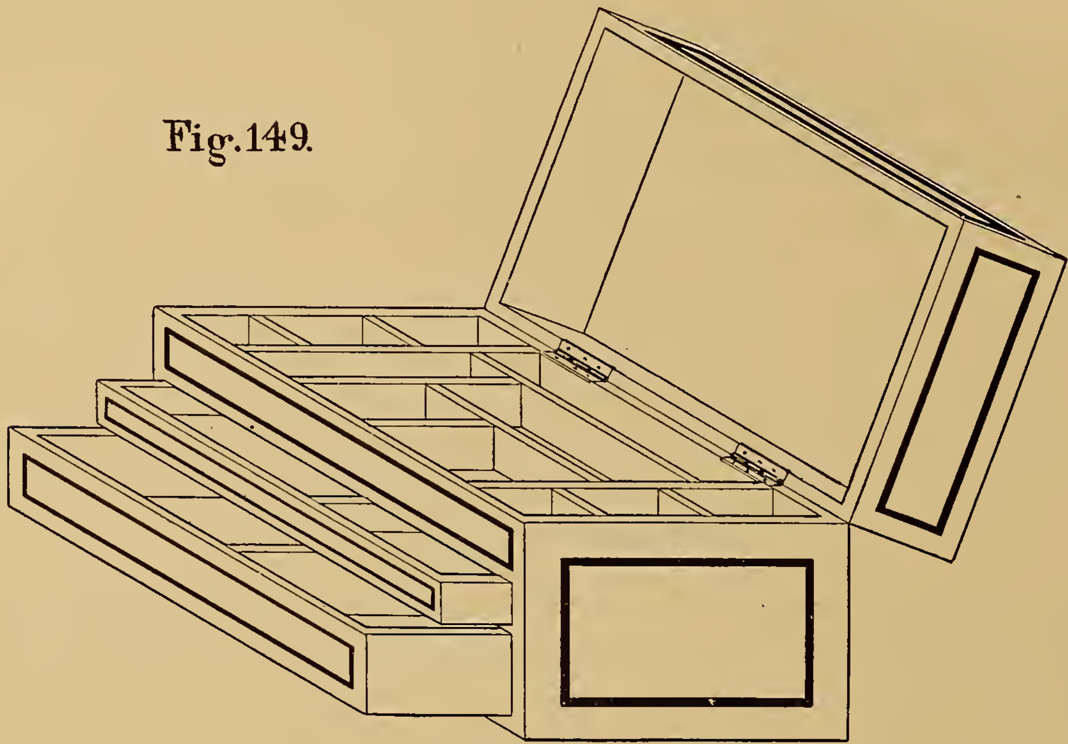
lines of their true length, but having any direction at pleasure; which enables us to show, in addition to the front face, either the right face or the left, the upper or the lower, as may best suit our purpose. And, as the figure shows, either of these faces at will may be made more conspicuous than the other by proper selection of the angles.

We have, then, a system of true *oblique projection:* it is more flexible than the isometric,

always quite as easily executed and in many cases more so, and like it exhibits the three dimensions in one view. All lines lying in planes parallel to the paper are shown in their true forms and relations; and not only these but lines perpendicular to the paper are shown of their actual dimensions, the introduction of any such senseless appliance as the "isometric scale" being prevented by the very nature of the process.

218. This system is well adapted for purposes similar to those in which isometric drawing

Fig. 149.



is employed,—such as the representation of joiner-work, as exemplified in the case of the box, Fig. 149, and in that of the peculiarly notched and fitted pieces shown in Fig. 150. In the

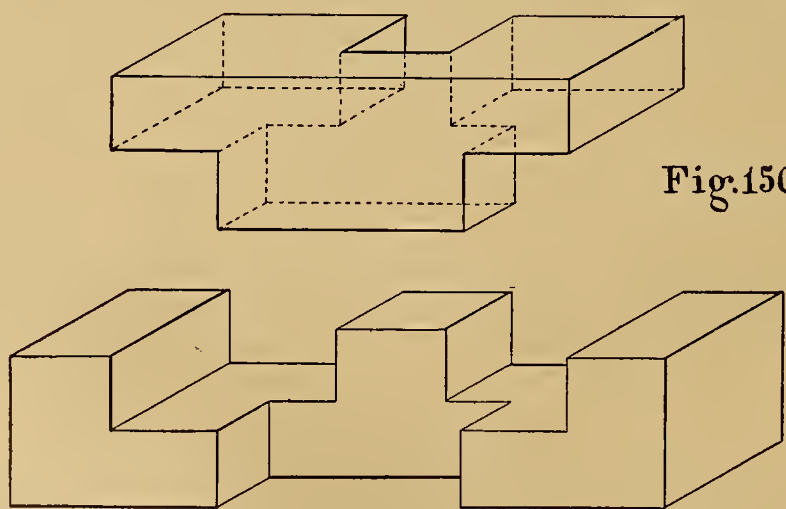


Fig. 150.

illustration, and especially in the sketching of small mechanical details, it possesses the decided advantage over isometry that, as shown in Fig. 151, circles whose planes are parallel to the paper are represented by circles, which greatly expedites the work of construction.

Those lying in planes perpendicular to the paper, however, must here too be represented by ellipses: since each circumscribing square is projected as a rhombus, the axes will coincide with the diagonals, and may be found as in Fig. 135.

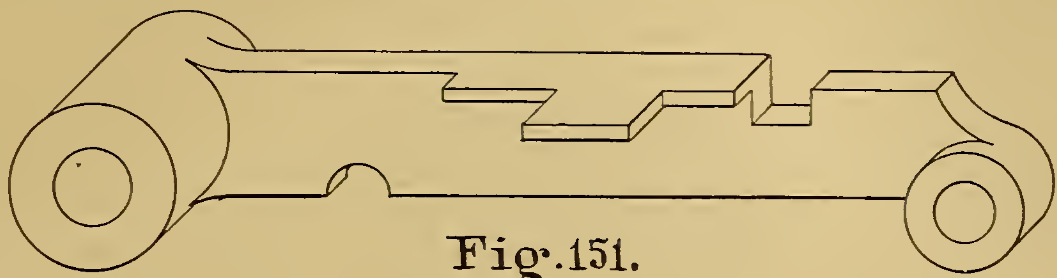


Fig. 151.

The use of ordinates, or offsets, in determining lines which are neither parallel nor perpendicular to the paper is substantially the same as in isometric drawing. Thus in Fig. 152

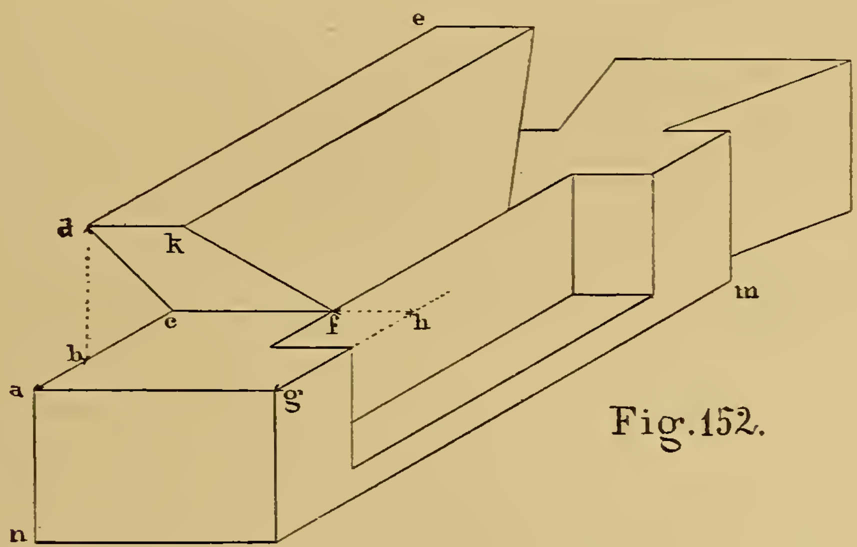


Fig. 152.

the point *f* in the plane *cg* is located by setting off *gh*, its distance from the plane *gn*, and then *hf*, its distance from the front plane *gm*; the point *k* is determined by *ab*, its distance from the end plane *gn*, *bd* its height above the plane *cg*, and *dk* its distance in front of the rear plane, which is invisible. The two points *f* and *k* being thus fixed, the projection of the line *fk* is determined; and the rest of the construction can be readily traced without explanation.

The lines which cast shadows, and are therefore to be made heavy, can usually be determined by inspection,—the light, as in the common orthographic projections, being supposed to come from over the left shoulder, and to go downward to the right as it recedes, in the direction of the body diagonal of a cubical room upon the wall of which the paper is hung, in front of the observer.

219. There is, then, no need to pursue this subject farther: the principles which have been thus briefly set forth are sufficient for applying either of these modes of projection to any subjects within the common range of practice. Both are very useful, with certain limitations which have been suggested, and the question whether either is suitable for any given case can be settled by experience alone.

But one thing has been decided by experience beyond all question; and that is, that the attempt to apply either isometric or cavalier drawing in the construction of a general plan of any complex machine is certain to result in a melancholy failure: the distortion, less noticeable in the case of minor details and detached pieces, becomes unendurable when the various parts are assembled.

PSEUDO-PERSPECTIVE.

220. For the purpose of producing a certain effect of relief, and conveying at least some idea of the three dimensions, while at the same time avoiding this distortion as well as the labor of constructing a true perspective drawing, a mode of representation has been devised,—to which the name of Pseudo-Perspective seems appropriate,—of which we give a single illustration in Fig. 153.

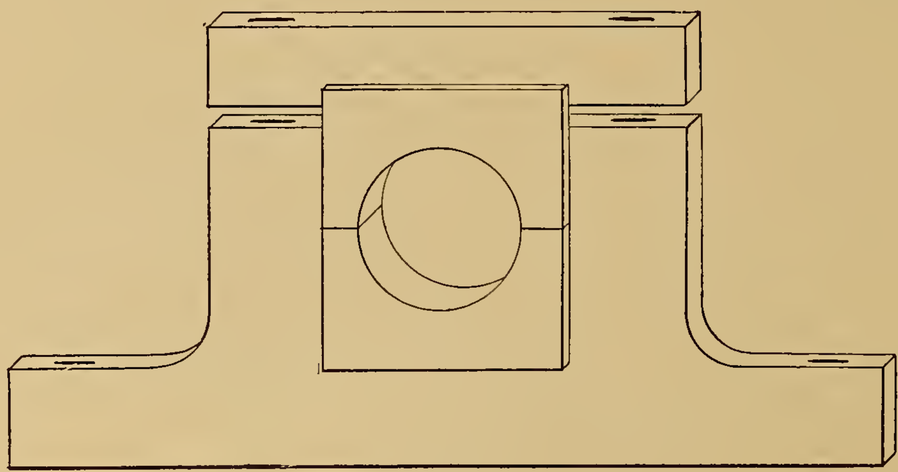


FIG. 153.

This is, in principle, a modification of cavalier projection; in that, as has been stated, the parallel projecting lines are inclined to the picture plane at an angle of 45°. But, referring to Fig. 147, it will be seen that if the cone of visual rays should have a less angle at the vertex, the projection of *ca* would be shorter; and by properly choosing this angle, the projection may be made shorter than the line in any desired ratio, while its direction is still entirely arbitrary.

The pseudo-perspective drawing, then, is made by representing the lines which are parallel to the paper in their true size, while those which are perpendicular to it are reduced to, say, one twelfth of the actual length, but parallel. This of course renders the result valueless as a working drawing; but it gives a sense of depth, and such representations are in many cases well suited for illustrations upon a small scale, such as cuts for encyclopædias and the like.

The distortion in the true cavalier projection is due to the mental recognition of the facts that the true representations of receding lines ought to converge, and that equal distances upon them ought to appear less as they recede. Both these errors are made less conspicuous by reducing the lengths of these representations, which is accomplished by the method of drawing above explained.

CHAPTER VIII.

THE SPUR-WHEEL AND THE BEVEL-WHEEL IN INCLINED POSITIONS. CONSTRUCTION OF THE CLOSE-FITTING WORM AND WHEEL. CONSTRUCTION OF THE SCREW-PROPELLER. STANDARD SECTIONING, ETC.

221. **It is assumed** that the student has now gained a knowledge of principles and modes of representation, and a degree of skill in the use of instruments, which will enable him to make presentable and accurate working drawings of mechanical details. But it is not proposed to give, as might be done, a collection of such drawings to be repeated as mere exercises.

Familiarity with the construction, and positive apprehension of the proportions, of any machine is best gained by actual examination of it; the copying of plans will never give an adequate idea of how the pieces would look when finished in accordance with them. The novice in designing is often much astonished when he sees in the solid the first piece for which he has made the drawing: he recognizes the form, of course, but it looks too large or too small, this part seems too light or that too heavy; and on the whole the physical appearance is somehow different from his mental conception of it. The fact is, let him who thinks he can, account for it as best he may, that there is a mysterious something in the relation between the drawing and the structure, the power to grasp which at once is born in few; and those who have it not by nature, will acquire it but slowly and painfully if they confine themselves to paper.

222. **The intention therefore is,** that the student should at once begin to acquire it in a more logical and practical manner. Instead of copying a plan, let him make measurements and sketches of some simple machine or part of a machine already built, and lay out a working drawing from the data thus obtained. Being thus obliged to take special note of the shape, size, and weight of the object in question, the impression upon his mind will be more distinct and lasting, and his drawing when done will convey to him a fuller meaning, than if the opposite course had been adopted; and besides, practice is had in a most essential acquirement.

The first efforts in this direction should be confined to comparatively simple subjects, of such dimensions that the drawings can be made **of full size**: the immediate object of this training is not to acquire skill in the construction of complex and difficult drawings, but to gain the power of forming, by inspection of any working plan, a correct conception of what it means. And experience has amply proved that this is best done by dealing at first with drawings of the same size as the objects themselves: whatever the reason may be, some little practice is usually needed in order to realize correctly in the mental image, the dimensions indicated by plans upon a reduced scale. Nature never acts *per saltum*, and this practice should progress by easy gradations.

223. Those who are left to instruct themselves must perforce devise for themselves the ways and means of making the measurements required; and if they succeed unaided, so much

the better for them,—just as it is better for a child thrown into the water to swim than to drown. But the best swimming-masters have decided that it does not pay to pitch applicants overboard and select classes from the survivors. Exactly so here: it is not the sole duty of an instructor, if there be one, to say "Sketch this," "Draw that," and then expect of his pupil a perfect result. If he knows how to show him how, it devolves upon him to do so if appealed to; and if the services of a thoroughly competent and experienced teacher are of value at any stage of progress, this is emphatically the one at which their value would be the greatest. And this not in regard to the sketching alone, but in the way of suggesting the best selection, and most important of all, the best arrangement, of the views which should be made in the working drawing.

In the second part of this work the matter of sketching is treated more at length, and hints in regard to arrangement will also be found in connection with the illustrations of that and other subjects.

We proceed, then, to the consideration of a few rather miscellaneous topics which may subsequently be found of service.

224. Drawing of a Spur-wheel in an Inclined Position.—In Fig. 154 we have an end view, a top view, and a section of a spur-wheel. Referring to Fig. 55 in relation to the forms of the teeth, it is sufficient to say that if the interior describing circle be of half the diameter of the pitch-circle the flanks will be radial lines, as here shown: the manner of drawing the top view and the section in this figure should require no explanation. Fig. 155 is a view of the same wheel, not in the direction of the axis as in Fig. 154, but at an angle with it. It is at once perceived that the pitch-circle WW will be projected here as an ellipse; and the points such as c, d, at which the teeth cut the pitch-circle, are projected to c', d' in the top view in Fig. 154, and thence to c'', d'', upon the ellipse $W''W''$ in Fig. 155.

Similarly, the tops of the teeth lie upon the outer circle of the wheel-blank, which also is seen as an ellipse in the inclined view; and so a, b are projected first to a', b', in Fig. 154, and thence to a'', b'' upon the corresponding outer ellipse.

If any circles be drawn between the pitch-circle and the outside one, they also will be seen as ellipses in Fig. 155, and by treating as above the points in which they cut the outlines of the teeth, as many intermediate points may be determined.

A like proceeding would enable us to draw the contours of the flanks were they curved as in Fig. 55. For the sake of convenience they are here made radial; for which reason they will in Fig. 155 appear as right lines converging to the centre of the ellipse $W''W''$.

All of the above relates to the appearance of the nearer side, or rather end, of the wheel. In regard to the farther one, it is clear that a pitch-circle scribed upon it would also be seen in Fig. 155 as an ellipse similar to $W''W''$, of which a portion is shown at $Y''Y''$; and to this all the preceding applies in a way too obvious to call for explanation. The consequence is that the visible edges of the teeth, such as those through a'' and b'', and the visible tangent elements, such as that through x, will be straight lines, of equal length, and of course parallel to the axis.

225. Drawing of a Bevel-wheel.—The basis of the bevel-wheel is a cone, vab, in the top view, Fig. 156. In the end view the base ab appears as a circle, which is called the *pitch-circle* of the wheel. The first step is to divide this circle into as many equal parts as the

wheel is to have teeth: this determines the *pitch*, measured by the arc *gi*, and the thickness *gf* is a little less than half the pitch.

Now draw at *a* and *b* lines perpendicular to *av* and *bv*; these meet the axis at *n*, and form the outline of a cone *nab*, normal to the pitch-cone. If this normal cone be developed, its base will become a portion of the circle whose centre is *n* and radius *nb*; an arc of which, *bgf*, is shown in the top view. Draw a radius of this arc at any point as *o*, on each side of which set off an arc, *of*, *og*, equal to half the arc *gf* in the end view. (Practically, it suffices to

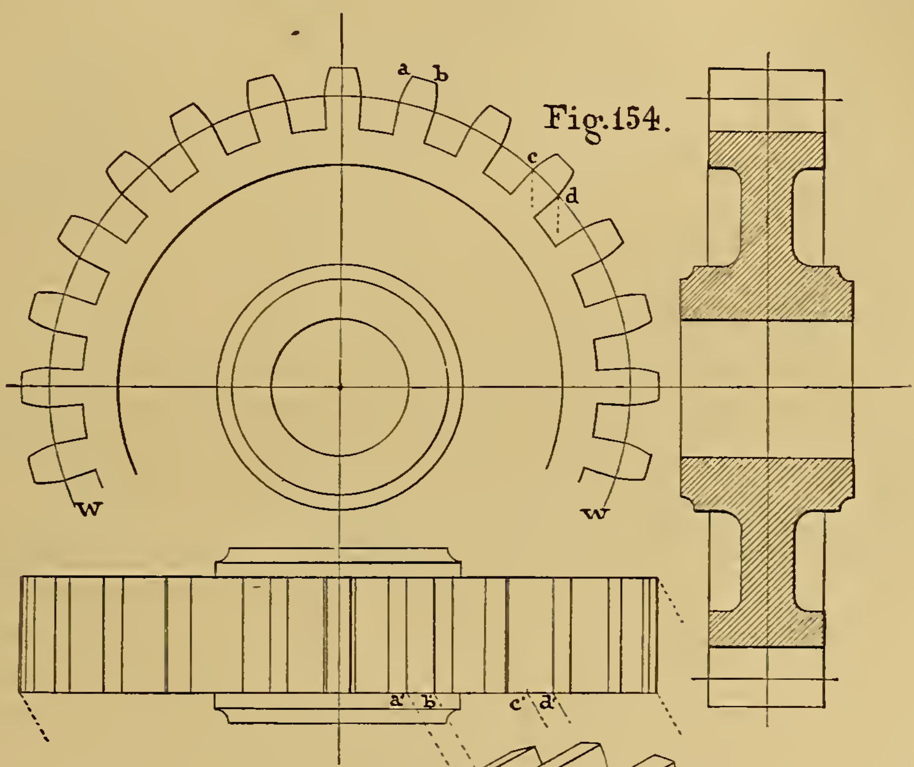


Fig. 154.

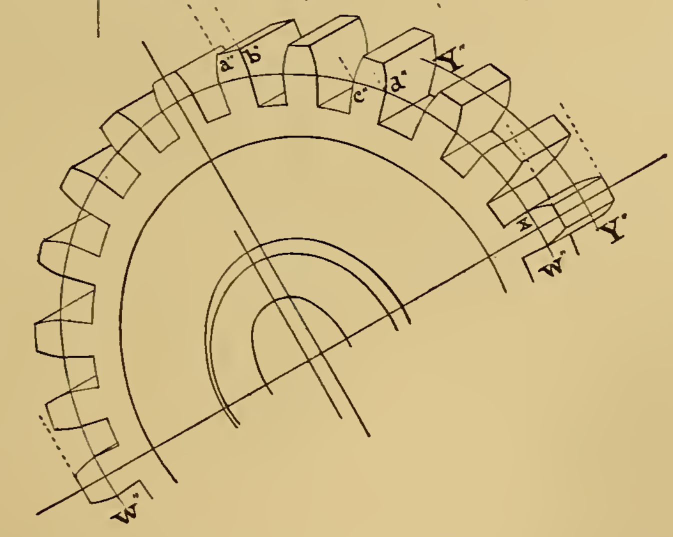


Fig. 155.

bisect *gf* and set off that distance on each side of *o*, because, unless the tooth is of unusual thickness, the difference between the arc and its chord is inappreciable.) Construct and proportion the outline of a tooth, regarding *bgf* as an arc of the pitch-circle of a spur-wheel. If this developed tooth were cut out of thin metal, and wrapped back upon the normal cone, its contour would be that required for the tooth of the proposed bevel-wheel. The doing of

this graphically, it will be seen, is an application of the process explained in connection with Fig. 105.

226. Having determined the height *rs* of the tooth, make *br*, *bs*, in the top view and in the section, respectively equal to *or*, *os*: then the tops and the bottoms of the teeth will be limited by the circles described by *s* and *r* in revolving around the axis.

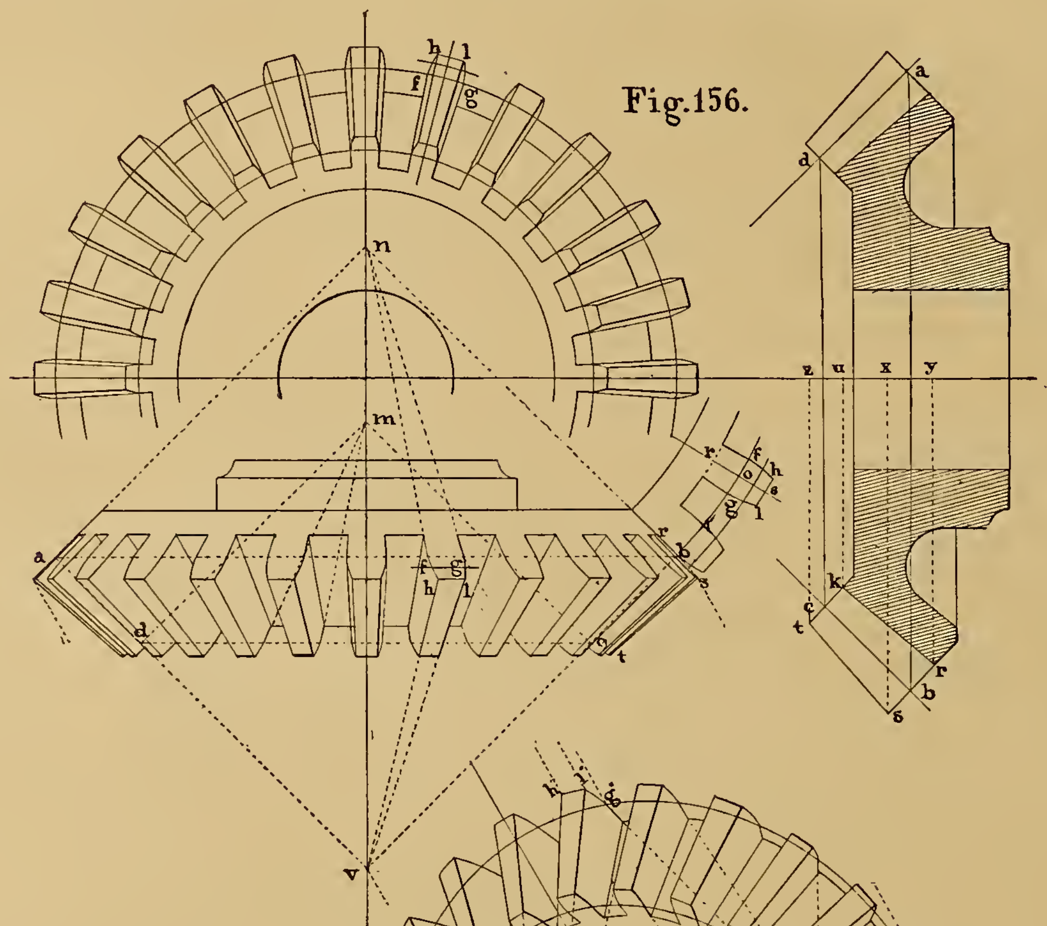


Fig. 156.

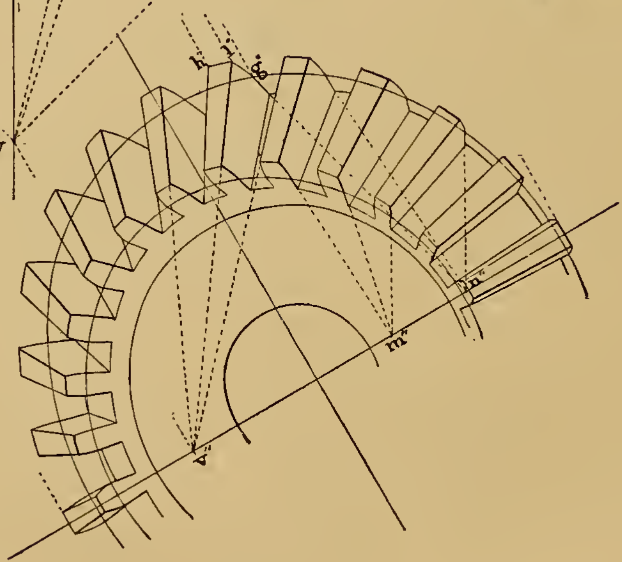


Fig. 157.

Next decide upon the length *bc* of the tooth in the sectional view, and draw at *c* a perpendicular to *bc*, also *rk*, *st*, converging to the vertex of the pitch-cone. It will be seen that the inner ends of the teeth lie upon another normal cone, *cdm*, in the top view, and that they will be of the same form as the outer ends, but smaller in the proportion of *tk* to *sr*.

The radius of the circle described by *s* is *sx*; upon this circle in the end view make *lh* equal to *lh* in the development: then the foreshortened view of the face curves will be *fh*, *gl*;

intermediate points in these curves may be found by means of intermediate circles, as indicated. The same method might be adopted for drawing the flanks, if they were curved; but since they are in this case made radial in the development, they will appear radial in the end view, and the apparent depth of the spaces between the teeth is limited by the circle whose radius is ry in the section.

The tops of the inner ends of the teeth lie upon the circle whose radius is tz, and their breadths may be determined by drawing radial lines as through l and h, the points limiting the breadth at the outer end; and these lines represent the edges of the teeth. So, too, radial lines through f and g cutting the inner pitch-circle whose diameter is cd, locate the junctions of the faces and flanks; since the latter in this instance are straight, they are also radial, and terminate in the circle whose radius is ku in the section.

227. One tooth having been laid out, all the others are copied from it in their proper positions in the end view. This is a necessary preliminary to the construction of the top view. In this, all the circles mentioned appear as straight lines, to which the points in which the tooth outlines cut these circles are projected from the end view, as shown at f and g on ab, h and l on the outer circle through s. Since the teeth are here made with radial flanks, the outlines of these will be straight, those upon the outer normal cone converging to its vertex n, and those which are visible at the inner ends of the teeth converging to m, the vertex of the inner normal cone.

It is thus quite evident that the correct drawing of the bevel-wheel is, if not a complicated, nevertheless quite a tedious, operation. Fortunately, this labor is not necessary for the purpose of simply having the wheel made: the shop drawing (Fig. 17, Part II) is far more easily made. But in a general plan of a complete machine, or for illustrative purposes, it may be necessary to draw such exterior views.

228. Drawing of a Bevel-wheel in an Inclined Position.—This involves no small amount of additional work, as will readily be seen by study of Fig. 157. But few words of explanation, however, are necessary, since the mode of operation is substantially the same as in Fig. 155. All the circles are now projected as ellipses, to which the various points of the tooth-outlines, as g'', h'', l'', are projected from the top view of Fig. 156. The vertices of the three cones appear in this new view as v'', m'', n'' upon the axis, and to these points the edges of the teeth, the flanks at the inner and those at the outer end, respectively converge.

229. Drawing of a Worm and Wheel.—The form of this device selected for illustration is that commonly called the close-fitting tangent screw: a steel screw of the same size as that finally to be used is notched to make a cutter, which is then hardened, tempered, and used for cutting the teeth of the worm-wheel. In this operation the cutter can remove only just so much metal as to permit of rotation: the ultimate fit of the driving worm is therefore as perfect as it can be made.

In Fig. 158 the diagram on the left is a section through the axis of the worm and perpendicular to that of the wheel; that is to say, by a plane LL in the diagram on the right, which is a section of the wheel by a plane passing through its own axis, and perpendicular to that of the screw.

Let C be the centre of the wheel and WPr a portion of its pitch-circle, to which draw TP tangent at P; and regarding this as the pitch-line, the next step is to lay out teeth as for

a rack and wheel: the outline of the rack will be the meridian section of the worm. The outlines of the rack-teeth might be composed of cycloids or other curves, but clearly it is easier to make a screw with a section bounded by right lines.

230. Fortunately, it is also better; and it may be done as follows: Draw the right line

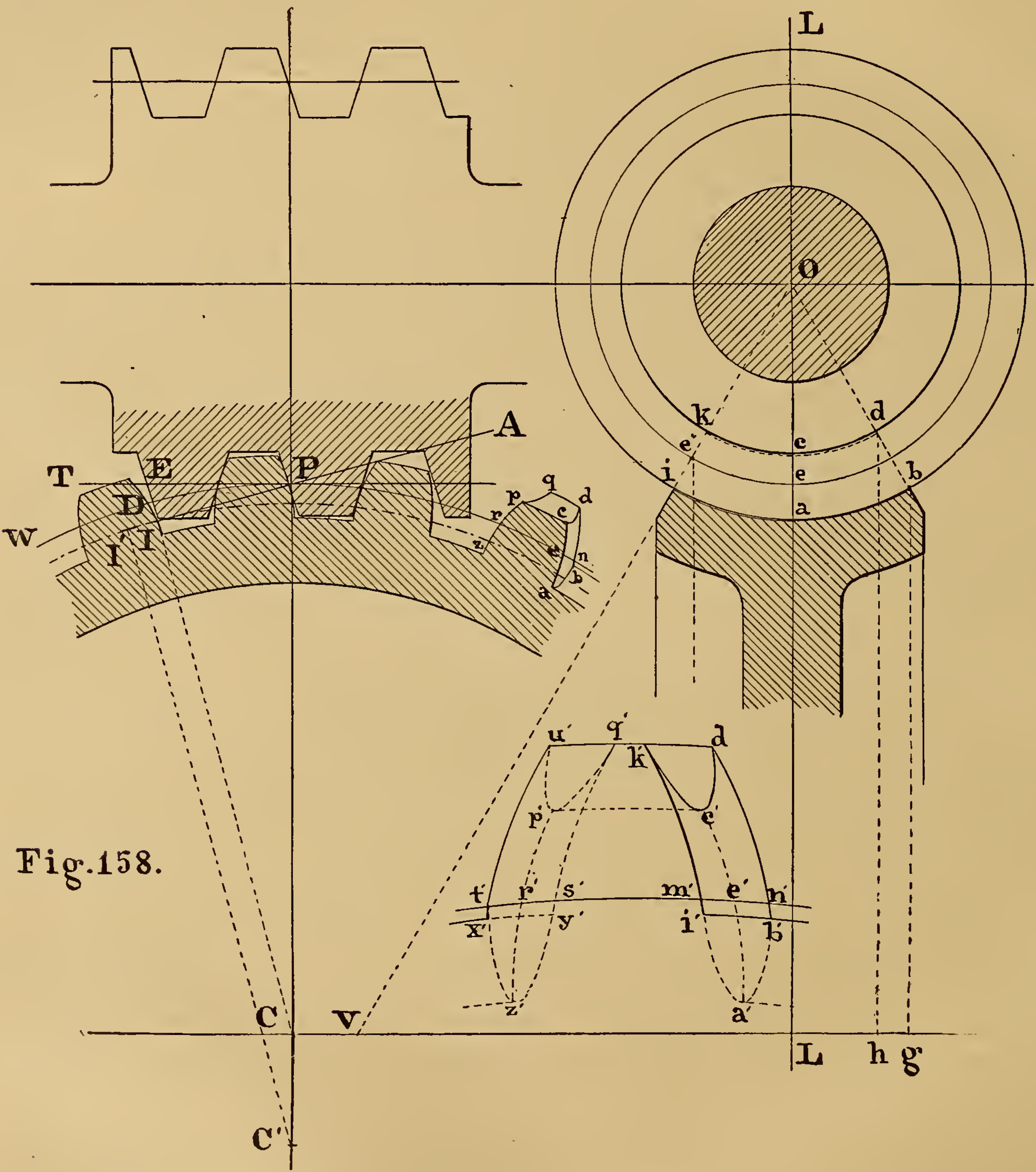


Fig. 158.

API, making an angle of from 15° to 17° with *TP*; let fall upon this the perpendicular *CI*, with which as radius describe about *C* the dotted circle shown. The sides of the wheel-teeth are involutes of this *base-circle*, and one side of the rack-tooth is perpendicular to *AI*; the other side of course being equally inclined in the opposite direction.

Divide the pitch-circle into as many equal parts as there are to be teeth in the wheel: this determines the *pitch-arc*, *PD*, in the diagram. On *PT* set off *PE* equal to this arc: then if, as here supposed, the worm is to be single-threaded, *PE* is the helical pitch. Practically, the projection of the rack-tooth below *TP* may be limited by a horizontal line either passing through *I* or a very little above it: this line then will be the lower outline of the cylindrical blank for the worm, of which the diameter may be from three to four times the pitch *PE*; when this diameter is decided the axis can be drawn; practically, also, the pitch-line *TP* may bisect the total height of the rack-tooth, which determines the bottom of the space and the diameter of the core of the screw. The tops of the wheel-teeth must just go clear of this core, and in like manner the spaces in the wheel must be deep enough to clear the outside of the worm: these spaces may be drawn with radial lines, within the base-circle.

The worm itself, then, is merely a **V**-threaded screw with a peculiar angle for the sides of the thread, and is easily drawn as explained in Chap. V; it is here shown in section only, for the sake of exhibiting the construction more clearly.

231. The fourth tooth in the diagram thus far considered shows not only the outline *aec* in the central plane *LL*, but also the extreme visible contour *abdqp*, bounding the tooth at the back, beyond the plane of the section. The determination of this contour requires an explanation of the right-hand diagram, in which *O* is the axis of the worm, the outer circle through *a* is the circumference of the blank, the inner circle through *c* is the circumference of the core, and the intermediate circle through *e* is the circumference of what may be called the pitch-cylinder, generated by the revolution of *TP* about the axis. The wheel is usually made, as here shown, to embrace about $\frac{1}{6}$ of the circumference of the worm: in this view, then, the lines *Ob*, *Oi* include an angle of 60°, and *Oi* produced cuts the axis of the wheel at *V*, and revolving about that axis generates a cone, which may be regarded as standing in the same relation to the worm-wheel that the normal cone had in relation to the bevel-wheel.

In applying analogous treatment, the argument would be as follows: *OV* represents the plane tangent to the cone, which cuts from the pitch-cylinder of the worm a right line, here seen as the point e'', and this line moves in rolling contact with the circle described by the point e'' of the wheel, in revolving about the axis *VL*. This circle is also the base of the normal cone, which would develop upon the tangent plane into an arc of a circle whose radius is Ve'': regarding this as a pitch-circle, the problem now is to find the form of a tooth which shall work with a rack whose form is that of the section of the screw by the same plane *OV*.

232. Fortunately, this is easily solved, because all the sections of the screw by planes through its axis are alike. Therefore, produce *PC*, making $PC' = Ve''$, and draw $C'I'$ perpendicular to *AI* produced: it will then be seen that the involute of a base-circle whose radius is $C'I'$ is the correct outline for the developed tooth. Or at least, if this were wrapped back upon the normal cone, the approximation to the required curve *bd* would be as close as is attained in the ordinary mode of laying out bevel-gearing.

But again, it is not necessary in any common case to take even this trouble, for two reasons: one is that this remote line is not a working line,—it is put in merely for effect and finish, and the cutter will make it right in metal however it is drawn; and the other is, that

this new involute will differ from the first one, within the limits used, so slightly that the difference will be inappreciable unless the work is on a prodigious scale. Practically, then, we may make this curve *bd* a portion of the same involute as the tooth-outline *aec*: we have now to determine its location and its limits.

233. The worm is a rack which advances by rotation; and one turn of it is equivalent to

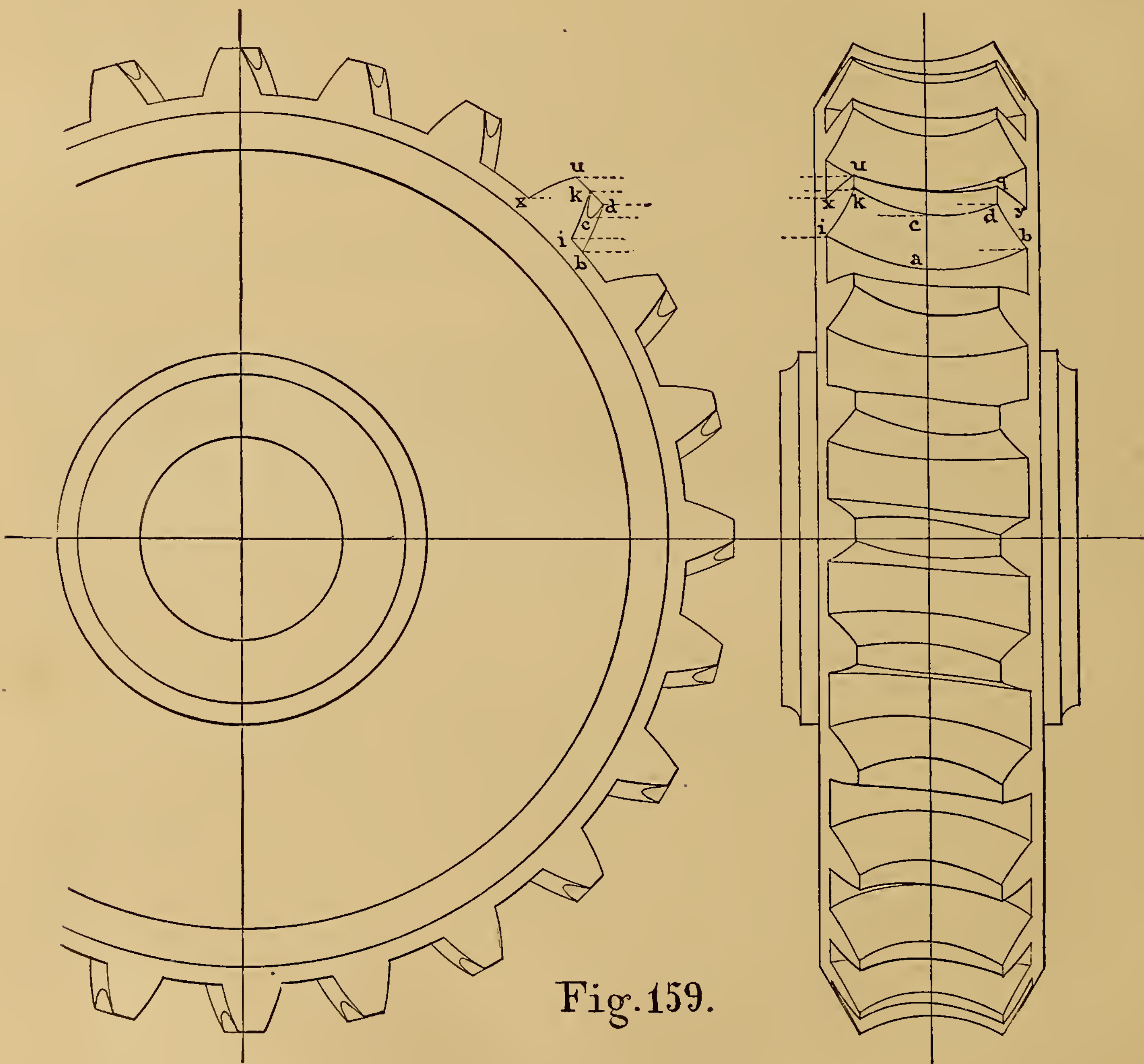


Fig. 159.

a longitudinal travel through a distance equal to the pitch. Now the angle *aOb* includes $\frac{1}{12}$ of the circumference; and in turning through this angle the worm would drive the wheel through $\frac{1}{12}$ of the pitch-arc. This fraction of the pitch, then, is set off from *e* on the pitch-circle *WPr*, thus locating the point *n*, through which the involute is drawn for the back of the tooth; which is limited at the top by a circle whose radius is *hd*, and at the bottom by one whose radius is *gb*, taken from the right-hand diagram; in which it is also seen that a curve

must extend from c to d, and another from a to b, each of which will be a sort of helix, lying however upon a concavo-convex surface instead of a cylindrical one.

An outside view of a complete tooth, greatly enlarged, is given below, where $t'n'$ is an arc of the pitch-circle, and the dotted outline $a'c'p'z'$ is the section of the tooth by the plane LL; $e'n'$ is $\frac{1}{12}$ the pitch, $b'n'd'$ the remote contour of the tooth. The contour on the nearer side of the wheel is a similar curve $i'm'k'$, $e'm'$ being equal to $e'n'$; and $k'c'd'$, $i'a'b'$ are the quasi-helical curves above mentioned. Points in these curves might be determined by drawing other normal cones inside of OVL, and repeating the process just described, by which k' and d' were located; but for ordinary purposes of representation this is needless, the limitation being already quite narrow: thus, for instance, the upper curve must be tangent to $p'c'$ at the point c'.

At the opposite side of the tooth $r's'$, $r't'$ are also each $\frac{1}{12}$ the pitch, and the same curves are drawn in the reverse direction, of which $x't'u'$ alone is visible.

234. Indeed, few of the curves above discussed are actually seen in the front view of the worm-wheel, as shown in Fig. 159. But it is necessary to determine them nevertheless if it is required to construct the side view, as will be clearly seen by aid of the letters, which correspond with those of Fig. 158.

All the teeth must first be drawn in the end view, as was the case with the bevel-wheel: the points of their outlines lie upon circles which appear in the side view as straight lines, to which they are projected one by one from the end view.

As in the cases of the spur-wheel and the bevel-wheel, it is wholly unnecessary for the purposes of construction to make these outside views of the worm-wheel, which involve so much labor: for illustrations of the working drawings for the shop, the reader is referred to Part II, Fig. 18.

235. Drawing of a Screw-propeller.—Fig. 160 shows the hub and one blade of a screw-propeller, of which three views are given, viz.: on the right, a view from the starboard side of the vessel, at the left of which is a view from aft, looking forward, and above this is placed the top view. The subject chosen for illustration is a *true screw*, that is, one whose surface is, like that of the familiar square-threaded screw, generated by a right line all of whose points describe true helices of the same pitch; and in this case it also remains always perpendicular to the axis: the pitch being assigned, any one of these helices is readily drawn. The element oo, in the central transverse plane of the hub, is assumed to be vertical: then the intersection aob of the acting face with the hub is determined, and the intersection awb of the back is assumed, as in Figs. 126 and 127.

This being done, the blade is otherwise limited as follows: A definite distance x, aft of the hub, being assigned, a plane pp is drawn perpendicular to the axis, which fixes the overhang of the trailing edge; also the radius of the small circle whose centre is 1 being given, an arc is described with that radius, tangent to pp and to po the outline of the cylinder whose diameter is that of the screw, and ac is drawn tangent to this arc. Also, a distance od is assigned, and bd drawn, to fix the overhang of the leading edge; an arc with given radius being now described, tangent to od and bd, the dotted line $ayeozb$ is the contour of a box, as we may say, within which the propeller is required to revolve.

236. The problem then is, to determine the intersection of the acting face with this peculiarly shaped surface of revolution.

Suppose the vertical element *oo* to advance without rotating to the position *uu* in the side view: it would then cut *ac* at the point *v*, whose distance from the axis is the same as that of *j*. But while thus advancing, the element in fact does turn, so as to assume the radial position *uu* in the end view; describe about *C* an arc with radius *Cj*, cutting this radius *uu* in *g*, which will be a point in the intersection. By repeating this process any number of points may be located, and the apparent contour of the blade fully determined. Since the ratio of

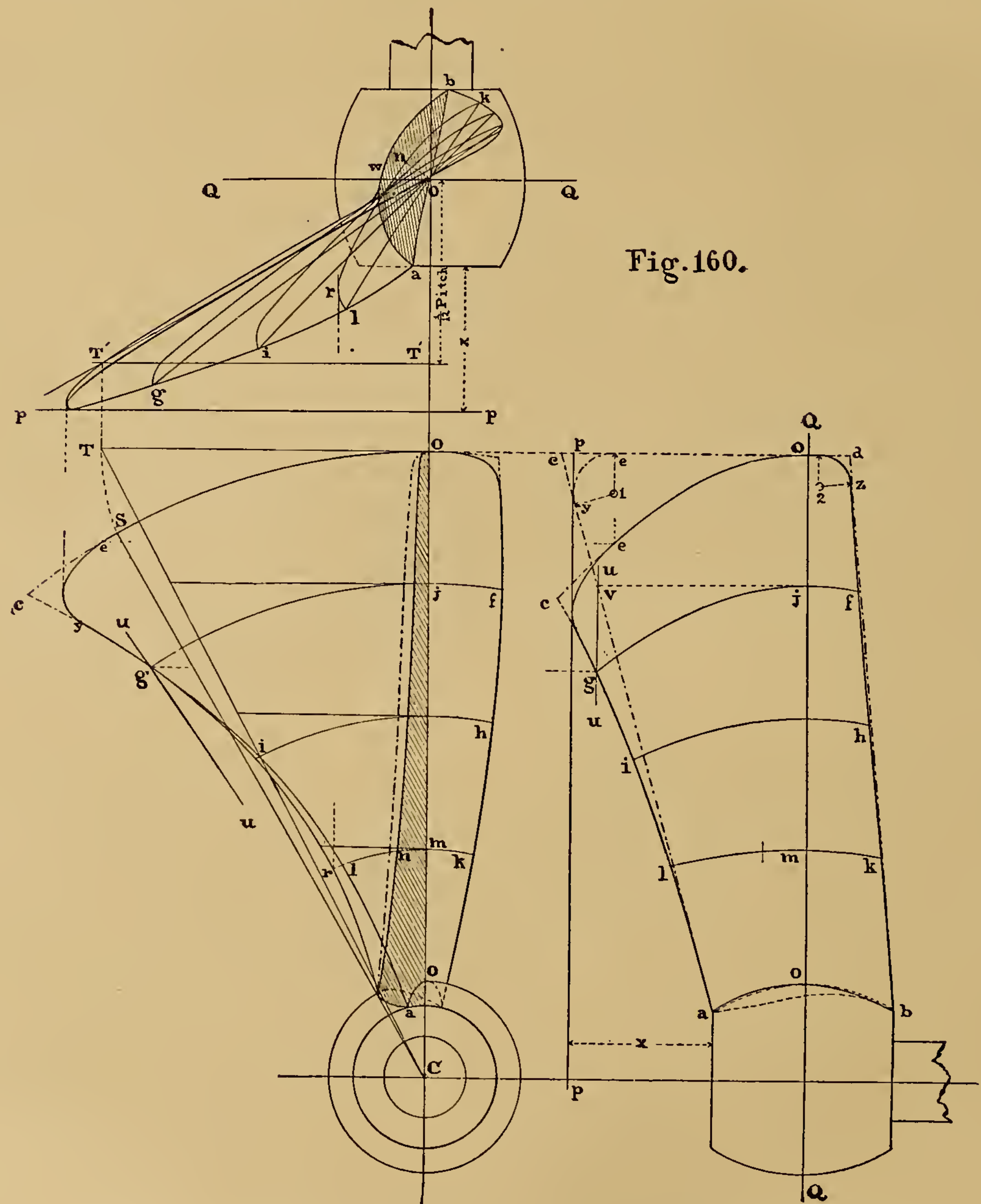


Fig. 160.

the rate of advance to that of rotation is constant, it will expedite matters to operate systematically, drawing a number of equidistant vertical lines in the side view and corresponding equidistant radii in the end view: a similar system of parallels in the top view will permit the projections of these curves to be worked out simultaneously in the three views, which is advantageous not only in saving time, but in serving as a check upon the accuracy of the construction, which is one of considerable delicacy.

237. In the top view the junction of the body of the blade with the hub is indicated by

sectioning, as in Fig. 127,—just as if the whole blade had been turned off in the lathe, leaving a mere film projecting. For the purpose of more clearly indicating the manner in which the thickness of the metal is gradually reduced as we recede from the centre and approach the thin edge at the periphery, intermediate sections of the blade are also shown in this view. These are sections, not by planes, but by cylinders—appearing in the end view as the circles *fg*, *hi*, *kl*, and in the side view as the correspondingly lettered helical arcs; in a word, they represent successive stages in the supposed process of turning off the blade. The sections of the acting face of the blade by these cylinders are therefore true helices, an aid to drawing which in the top view will be found in first determining the tangents at *o*. A convenient method of doing this is to draw at *o* in the end view a tangent to the periphery, and upon it to rectify a convenient fraction *oS* of the circumference, thus determining *oT*: in the present case the fraction selected was one twelfth. Then in the top view draw a parallel to *QQ*, at a distance from it equal to the same fraction of the pitch; to this line *T* is projected from the view below, and *To* thus determined is tangent to the outer helix. In the end view draw *CT*, and intersect it by tangents to the other circles, *fg*, etc., by which the sections are made: the points thus found, projected to *T′T′* in the top view, will be like manner lie upon the tangents to the other helices.

238. These cylindrical sections then are bounded in front, that is to say, on the acting face, by helices. The lines bounding them on the back cannot be determined by any fixed rule: they are in fact commonly drawn arbitrarily to a great extent, the leading edge being always, however, made thinner and sharper than the trailing edge, because in the main the propeller is used for going ahead and not for backing. But as a further guide to the distribution of the metal, it is a common practice to put in the end view, as shown in full lines, what is called a "conventional section." This is not a true section by the plane *QQ*, however, which would give to the back of the blade a form like that shown in the dotted line: the conventional section indicates the required *normal* thickness of the metal at points upon the vertical element *oo*. From this we may derive a little assistance in constructing the back lines of the cylindrical sections; thus, for instance, the circle *kl* cuts *oo* at *m*, and the line of the conventional section at *n*; in the top view set off *On* equal to *mn*, in a direction normal to the helix *lok*: then *n* is one point in the corresponding back line *lrnk* of this particular section, and a like determination is to be made for each of the others.

239. In the end view the contour of the leading edge is the curve *khf*, beyond which nothing is visible. The outline of the trailing edge is the curve *aligy*; but the thickness of the blade at its junction with the hub and for some distance outward is so great that the back, it is obvious, can be seen beyond the trailing edge. And the cylindrical sections will enable us to ascertain its contour: for example, draw parallel to the axis a tangent to the curve *knl*; this line is an element of the cylinder, and touches the curve at some point *r*, which is projected to the circle *knl* in the end view, thus locating a point in the required visible outline of the back; as many more as desired may be found in like manner.

240. Since the back lines of the cylindrical sections are, substantially, assumed, it must be admitted that there is no absolute certainty that the surface of the back will be "fair," that is, free from humps or hollows. To make certain of that, it would be necessary to make such sections quite close together, and also a series of sections by transverse planes,

also close together, and to alter the lines until the two sets were not only concordant, but composed of fair lines throughout. This, however, it is needless to do unless a *pattern* is to be made in exact accordance with the drawing. Such propellers are usually made by "striking up" the face in loam; this done, a sort of pattern in sand is constructed upon it, the back of which the moulders fashion with their trowels: this is used to make a mould for the back of the blade, and then destroyed.

When this course is pursued, a drawing such as has been described serves every purpose.

241. The appearance of the complete propeller with four blades is shown in Fig. 161. In

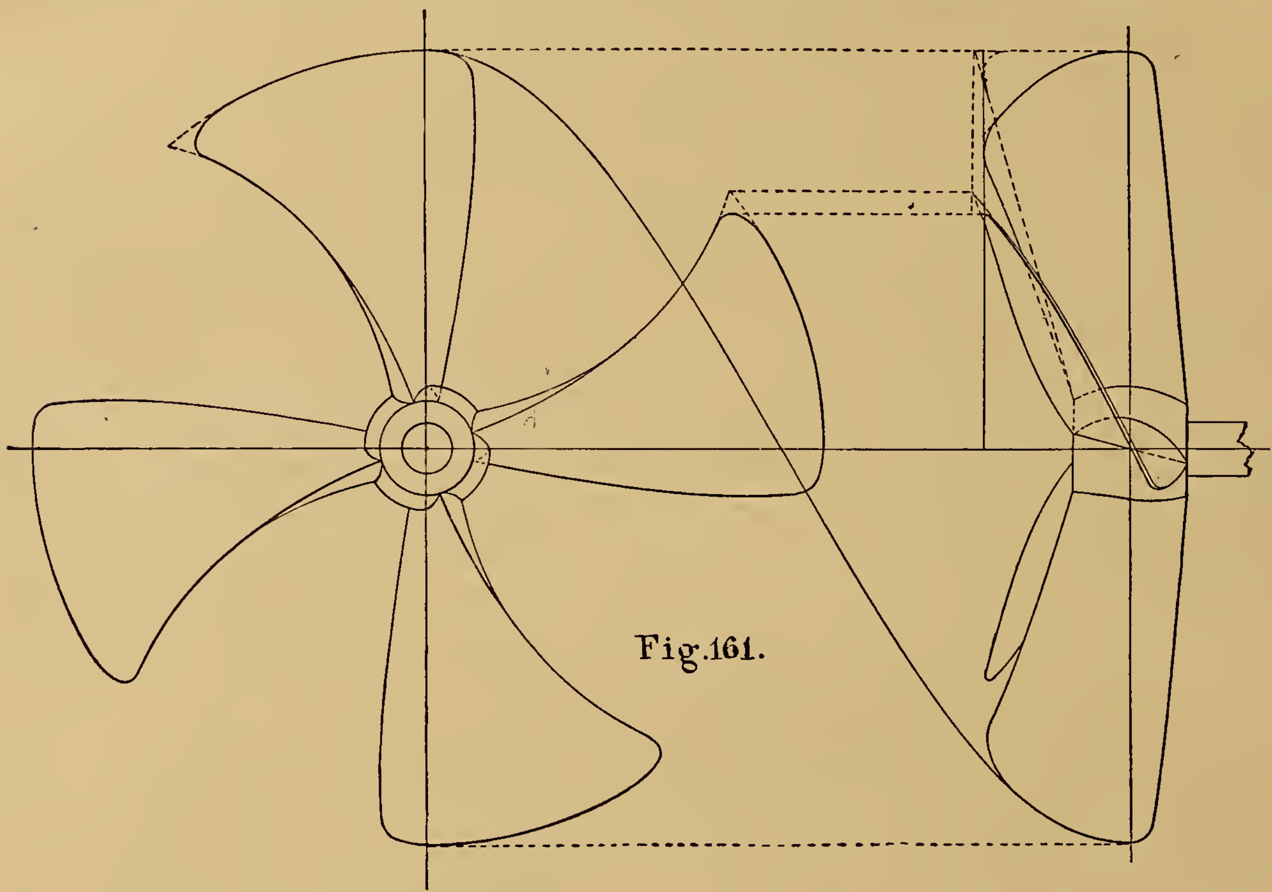


Fig. 161.

relation to this it may be said that the placing of the two views at such a distance apart as to include between their vertical centre-lines just one half the pitch, is rather fanciful perhaps. Still, when the proportions are such as to make it convenient, the opportunity thus afforded to represent in an appropriate position one half of the outer helix may well be utilized to convey graphically an idea of the proportion between the pitch and the diameter of the screw,—which can hardly be done in a more effective manner than that here adopted.

242. In conclusion, a few words may be said in regard to certain modes of sectioning. It is by some laid down as an absolute rule that the section-lines should be inclined to the outlines at an angle of 45°. But if this be adhered to in the case of a long and thin piece, the effect, as any one can satisfy himself by trial, will be very unsatisfactory; an angle of 30° with the longer side will produce a much more pleasing effect, and will, besides, require fewer lines and consequently less time.

In the representation of work composed of rolled plates, such as boilers, the details of

iron ships, and the like, the most effective as well as the most expeditious method is to rule the sections with fine longitudinal lines, as shown in Fig. 162.

But if such work is to be drawn upon a small scale, it will often happen that the attempt to represent the thickness of a sheet of metal by even a double line is not only laborious, but unsuccessful. In such cases a very telling and comparatively easy expedient is shown in the lower part of 162: the whole area of the sectioned parts is filled in with solid black, care being taken to show the joinings by leaving a fine line of pure daylight between the surfaces which are actually in contact. Fig. 163, which exhibits the side armor of an iron-clad vessel of the Monitor type, is an example of the effectiveness of this style for certain purposes in producing emphatic contrasts and making conspicuous the important features. But, obviously, contrasts so violent as this are suitable only for drawings made on comparatively small scales.

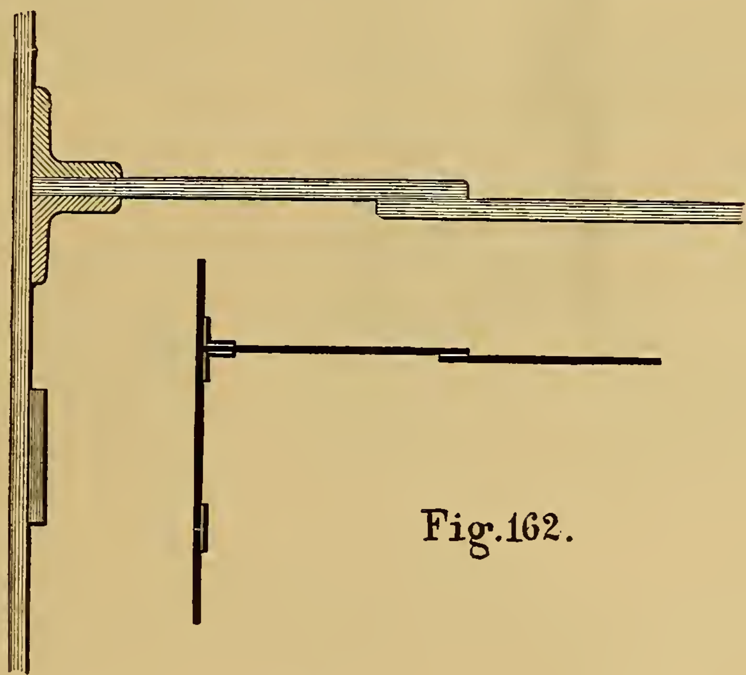


Fig. 162.

243. It was stated in (**15**), that in sectioning the aim should in general be to imitate the effect of an even tint covering the surface, so transparent as not to obscure the dotted outlines of parts beyond, and therefore of necessity too light to impair the distinctness of the lines defining its own boundaries. It is not imperative that the spacing should be the same throughout: if any pieces are small they may be, and if quite small they must be, sectioned more closely than the larger ones, though with lines a trifle finer; nor will this make them unduly prominent, nor destroy the general effect of uniformity in the tone. This of course has reference to the finish only, and is upon the supposition that the ultimate object is to produce the most agreeable effect; the fact that certain portions are cut is distinctly shown, but no information as to the materials of the structure is conveyed.

That information is of course essential to the completeness of a working drawing, and it may be conveyed in more than one way. In the days when, before the invention of the blue-print process, tracings were sent into the shop, one mode of indicating the different substances was by the use of colors, wrought-iron for instance being sectioned in blue, cast-iron in black,

composition metal in red, and so on. This had the advantage of making the distinctions clear and conspicuous, without impairing the even and finished effect.

244. Colors being inadmissible when blue-printing or kindred processes are employed, the practice has obtained to some extent of sectioning different materials in different styles, with an effect upon the appearance of the drawing as a whole which in most cases it would be gross flattery to call hideous. An effort has been made to secure general adoption of some uniform system or standard in regard to this, but as yet without positive results. In Fig. 164 we give a few specimens selected from the series arranged by Mr. Frank Van Vleck in

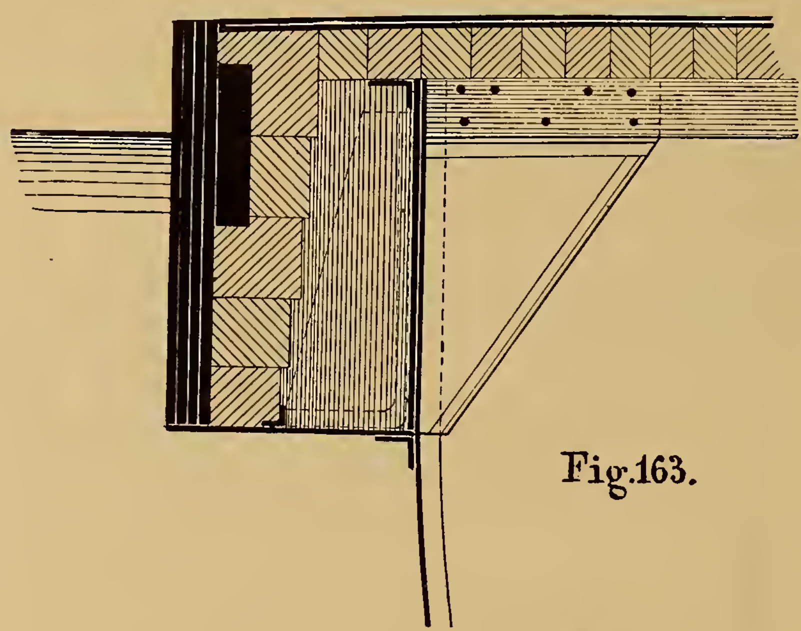


Fig. 163.

furtherance of that effort;*—a series probably as perfect and as free from objections as any that could be devised. The materials are as follows:

A. Wrought-iron.	F. Brick.
B. Steel.	G. Cast-iron.
C. Wires.	H. Wood.
D. Stone.	I. Babbitt-metal.
E. Brass.	K. Vulcanite.

Aside from the ruinous effect of this method upon the appearance of the drawing, which may be regarded by many as of slight importance, it appears open to one or two serious practical objections. It is not easy to do the sectioning well in such styles, for instances, as those shown in *A*, *B*, *E*, and *K*, and it is impossible to do it rapidly, which is in many cases of considerable importance; again, absolute dependence upon any system would demand perfect familiarity with the adopted code of symbols on the part of all the workmen for whose guidance the drawings are made.

* See *American Machinist*, April 9, 1887.

245. This last would apply with equal force to the use of colors, and indeed it appears in any case advisable to make use of reference-letters, of which an explanation should be written upon each sheet, as, for example,

a, *a*, Cast-iron ; *b*, *b*, Wrought-iron ; *c*, *c*, Steel, etc. ;

and then let each piece be marked with the proper letter, according to the material of which it is to be made. This course, which is both simple and certain, was extensively used even

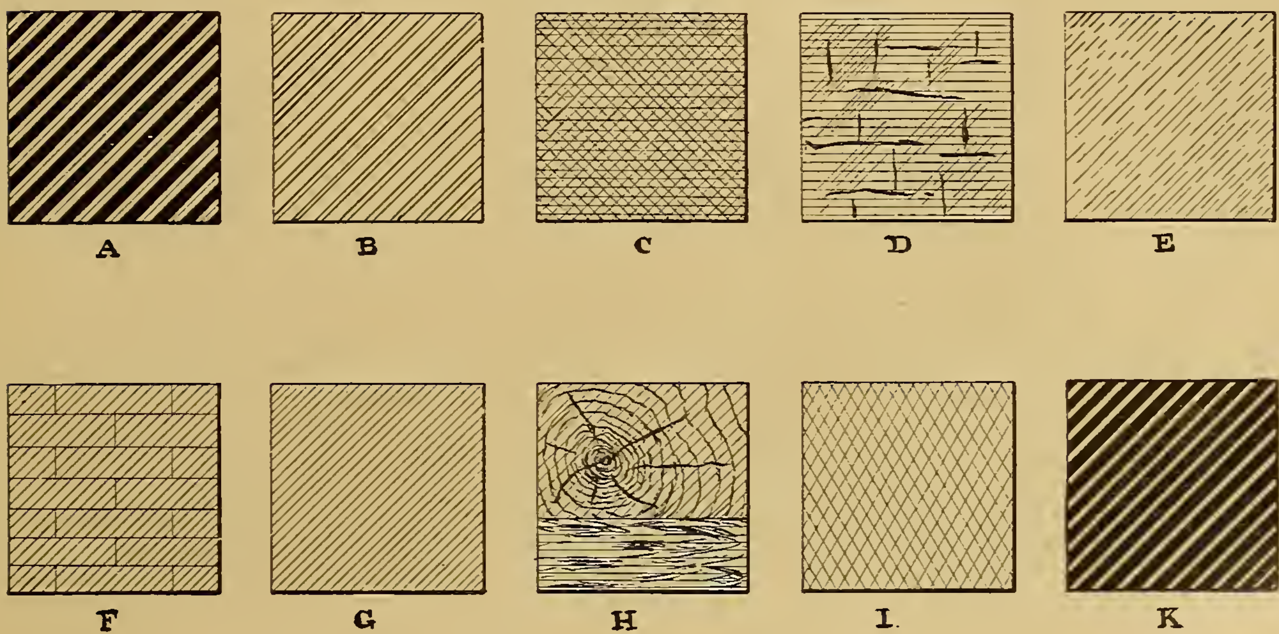


Fig. 164.

when the sectioning was done in colors : it is equally applicable if the whole drawing is made in plain black, thus enabling the draughtsman to avoid the unpleasantly variegated effect of the proposed standard sectioning, and to produce results rivalling in harmony of tone those of a well-executed engraving.

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